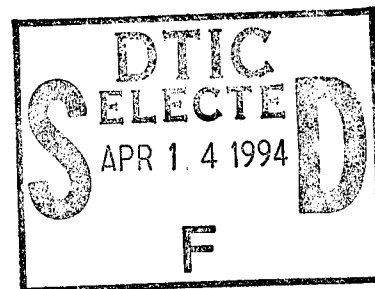


# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



## THESIS

FAILURE ANALYSIS OF A REPAIRABLE SYSTEM:  
THE CASE STUDY OF A CAM-DRIVEN RECIPROCATING  
PUMP

by

Donald D. Dudenhoeffer

September, 1994

Thesis Advisor:

D.P. Gaver

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THE CASE STUDY OF A CAM-DRIVEN RECIPROCATING PUMP

by

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Submitted in partial fulfillment  
of the requirements for the degree of

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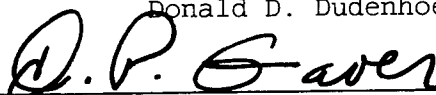
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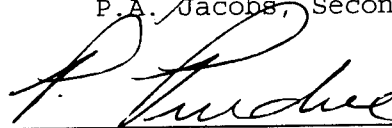
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## ABSTRACT

This thesis supplies a statistical and economic tool for analysis of the failure characteristics of one typical piece of equipment under evaluation: a cam-driven reciprocating pump used in the submarine's distillation system. Comprehensive statistical techniques and parametric modeling are employed to identify and quantify pump failure characteristics. Specific areas of attention include: the derivation of an optimal maximum replacement interval based on costs, an evaluation of the mission reliability for the pump as a function of pump age, and a calculation of the expected times between failures. The purpose of this analysis is to evaluate current maintenance practices of time-based replacement and examine the consequences of different replacement intervals in terms of costs and mission reliability. Tradeoffs exist between cost savings and system reliability that must be fully understood prior to making any policy decisions.

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## **EXECUTIVE SUMMARY**

Today's Submarine Force is faced with many new challenges. These include an ever-expanding role in Joint and Maritime operations, despite a diminishing operating budget and a shrinking force level. Submarine maintenance is one key area that must be carefully evaluated to ensure maximum operational readiness while working within the confines of budgetary constraints. A Maintenance Effectiveness Review, MER, is currently in progress to evaluate the applicability and effectiveness of all aspects of submarine maintenance. Any comprehensive evaluation of maintenance practices, however, requires a detailed analysis of equipment performance in terms of failure characteristics and life expectancy.

This thesis supplies a statistical and economic tool for analysis of the failure characteristics of one typical piece of equipment under evaluation: a cam-driven reciprocating pump used in the submarine's distillation system. The purpose of this analysis is to evaluate current maintenance practices of time-based replacement and examine the consequences of different replacement intervals in terms of costs and mission reliability. Tradeoffs exist between cost savings and system reliability that must be fully understood prior to making any policy decisions.

The analysis examined the failure characteristics of a sample set of 61 pumps installed between May 1987 and December 1993. The data consisted of 140 failures, of which 14



required pump replacements, and 2020 total months of pump operation. The primary source of the data is the Navy's 3-M system. Pump failures are classified as repairable or non-repairable and thus require replacement. The probability that a given pump failure is repairable was evaluated to be 0.9. Additionally, a new pump will experience on average 10 failures cycles prior to being replaced.

Trending analysis indicated an increasing failure rate with pump age; a sign of system wearout. The increasing failure rate, likewise, indicated that a time-based replacement maintenance policy may be warranted. A Nonhomogeneous Poisson Process (NHPP) was chosen to model the pump's failure characteristics. Maximum likelihood was then used to estimate the NHPP's parameters.

The use of a stochastic modelling allowed a comprehensive evaluation of the current maintenance policy of time-based replacement of the pump at a periodicity of 36 months for Trident submarines and 60 months for non-Trident submarines. The pump is the same for both platforms. The optimal replacement interval based entirely on minimizing the long-run average system costs was determined to be 111 months for an average lifecycle cost of \$2226 per month. This periodicity, however, resulted in undesirable mission reliability. The resulting probability of the pump completing a mission of three months just prior to replacement was only 0.23 with 1.47 expected pump failures during the mission. A replacement periodicity of 36 months resulted in a mission reliability of 0.76 with 0.27 expected failures at an average lifecycle cost of \$3674 per month. The 60 month periodicity had a mission

reliability of 0.63 with 0.46 expected failures at an average cost of \$2611 per month.

The results of this analysis indicate that a 60 month replacement interval may be acceptable based upon reliability requirements. The periodicity of 36 months, however, may be excessive and warrant extension to 60 months. This thesis does not attempt to quantify any minimum reliability requirements. It is evident, though, that a replacement schedule based strictly on economic considerations is unsatisfactory from a reliability standpoint. This illustrates one of the most powerful uses of stochastic modelling: the ability to predict and evaluate the consequences of maintenance policy decisions on system performance.

The scope of this thesis has implications beyond the operation of this one pump. This case study of the cam-driven reciprocating pump illustrates the type of analytical techniques necessary to perform a comprehensive evaluation of a shipboard system's performance. The goal is to provide decision-makers with the in-depth statistical foundation to make sound decisions on maintenance policy; decisions that directly affect the readiness and ability of the U.S. Submarine Force to assume its expanding mission.

## INTRODUCTION

### A. BACKGROUND

The end of the Cold War and growing concerns over the national debt have forced the Navy, and specifically the Submarine Force to reevaluate its current operational policy. One of the areas being closely evaluated is that of Submarine Maintenance. RADM (sel) R.E. Frick, NAVSEA Deputy Commander (Submarines Directorate), recently directed an evaluation of all submarine maintenance performed at Depot (shipyard), Intermediate Maintenance Activity (IMA), and Ship's Force levels. This Maintenance Effectiveness Review (MER) is a continuous process to evaluate the maintenance needs of the Submarine Force while working within the fiscal constraints of a shrinking budget. NAVSEA PMS390, Submarine Monitoring, Maintenance, and Support Program Office (SMMSO) is the lead activity in charge of coordinating this review.

The review process requires an in-depth review of the applicability and effectiveness of all maintenance actions performed by the submarine force. The goal is to eliminate redundant and ineffective maintenance while maintaining

optimal materiel readiness, within a limited budget. The concepts of Reliability-Centered Maintenance are being used to evaluate the time-based and "fix when fail" policies historically used by the Navy.[Ref.1] The key to a successful shift in strategies is the ability to monitor equipment conditions and make reliable maintenance decisions based on real-time data and predictive analysis.

The goal of this thesis is to address the analytical needs of the Submarine Force necessary for a thorough assessment of current maintenance policies.

## **B. CURRENT ANALYSIS**

The SMMSO organization consists of a staff of approximately one hundred personnel based in Washington, D.C., as well as monitoring teams stationed at each submarine base. The focus of SMMSO's work is submarine equipment performance monitoring and maintenance recommendations. On a periodic basis, SMMSO site teams perform various performance tests on a submarine's systems. These tests involve everything from the vibration monitoring of a pump to oil analysis of shaft lubricating oil. The results of these tests, as well as any significant information on past problems, are analyzed by a specific engineer who is SMMSO's system expert on that equipment. This

information is used to predict future equipment performance, make recommendations on corrective actions, and make maintenance deferment and equipment replacement decisions.

In the past, the scope of the statistical analysis was focused on short-term predictions, i.e., refit to refit, which was at the time, adequate to formulate maintenance policy given the large Defense Budget and high priority on submarine readiness. These policies have been very conservative in nature and possibly more restrictive than necessary. In view of the shrinking budget, these policies should be reviewed to ensure the optimal use of available resources.

### **C. PROBLEM STATEMENT**

The specific area of analysis to be addressed in this thesis is the performance, from the standpoint of reliability, availability, maintenance, and replacement needs of a particular *cam-driven reciprocating pump*. The Class Maintenance Plan (CMP) of this pump is currently being evaluated for effectiveness and applicability. Mr Richard Youngk is the SMMSO systems engineer conducting a performance analysis of the *cam-driven reciprocating pump*. This thesis is in conjunction with his efforts and builds upon his research and analysis. [Ref.2]

The selection of this particular pump for the study is in itself not significant, but the pump is typical of the systems being analyzed. The analysis presented in this thesis focuses in a broader context on the specific questions being asked more broadly. This thesis proposes a process or method to systematically analyze the pump data presently available. It is believed that this procedure can then be adapted for use to analyze other systems.

Specific questions to be addressed in the analysis are:

1. What is the expected number of pump repairs and replacements required over a certain time interval?  
What is the associated expected repair cost and downtime?
2. Does the failure rate change over the life of the pump?
3. What is the optimal equipment replacement interval?
4. Can a useful stochastic model of pump performance be constructed?

#### **D. FAILURE**

Before proceeding further with the analysis, it is necessary to define what is meant by the term: *failure*.

A failure is classified as an event or inoperable state, in which any item or part of an item does not, or would not, perform as previously specified.[Ref.3] This paper further classifies failures as being either *repairable* or *non-repairable*. A repairable failure implies that the system can be returned to operating specification with the replacement of only a small fraction of the system's parts and in a short period of time. Non-repairable implies the pump must be completely replaced, or else an extensive overhaul is necessary to restore pump operation within allowable limits. In the context of this paper, pump replacements are limited to replacements resulting from a non-repairable failure and not replacements based upon any other criteria.

#### **E. FAILURE MODELS**

Two basic models often used in evaluating repairable systems are the Homogeneous Poisson Process (HPP), and the Nonhomogeneous Poisson Process (NHPP). Both the HPP and NHPP are counting processes used to model the number of component or equipment failures occurring over the life of the equipment. Figure 1.1 shows the life-cycle of a piece of equipment, with  $t_i$  representing the time to the  $i^{\text{th}}$  failure, and  $x_i$  representing the time interval between the  $(i-1)^{\text{st}}$  and

$i^{\text{th}}$  failures, i.e.  $x_i = t_i - t_{i-1}$ . To be more explicit,  $x_{ki}$  represents the time from the  $(i-1)^{\text{st}}$  to  $i^{\text{th}}$  failure for item (here pump)  $k$ . Replacement of pump  $k$  means that the repairable-failure generating Poisson process starts again from scratch.

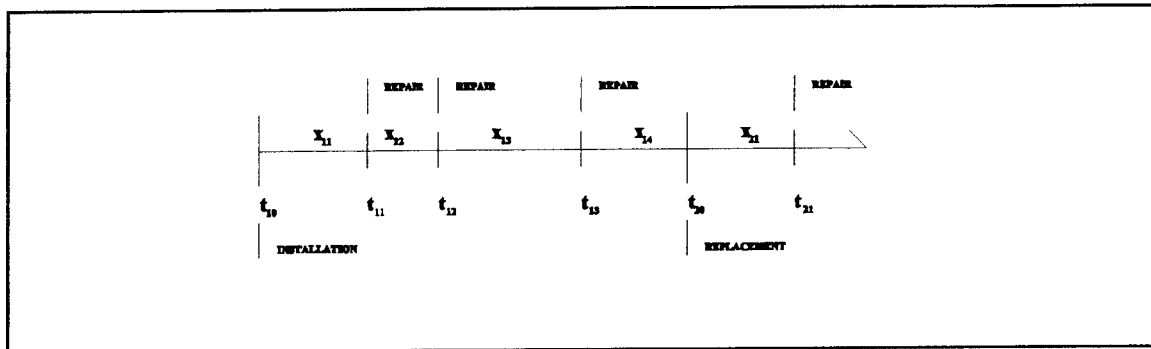


Figure 1.1: Model of Pump Life

Now let  $N(t)$  equal the number of failures occurring prior to pump age  $t$ . Then  $N(t)$  is considered a counting process,  $\{N(t), t \geq 0\}$ , if the following conditions hold:

1.  $N(t) \geq 0$ .
2.  $N(t)$  is an integer.
3. If  $s < t$ , then  $N(s) \leq N(t)$ .
4. For  $s < t$ ,  $N(t) - N(s)$  represents the number of events occurring in the interval  $(s, t)$ .



## 1. Homogenous Poisson Process

The Homogenous Poisson Process is a special counting process which is commonly used to describe systems with a constant failure rate  $\lambda$ . More precisely, the counting process,  $\{N(t), t \geq 0\}$ , is said to be a Homogenous Poisson Process with rate  $\lambda$ ,  $\lambda > 0$  if:

1.  $N(0) = 0$ .
2. The process has independent increments, i.e., the number of events occurring in disjoint time intervals are independent.
3. The number of events in any interval of length  $t = \tau_j - \tau_i$  is Poisson distributed with mean  $\lambda t$

i.e. for  $n = 0, 1, 2, \dots$ ,

$$Pr[N(\tau_j) - N(\tau_i) = n] = e^{-\lambda(\tau_j - \tau_i)} \frac{(\lambda(\tau_j - \tau_i))^n}{n!} \quad (1.1)$$

$\tau_i < \tau_j, n = 0, 1, \dots$

Furthermore, the conditional probability that the system will survive to time  $\tau_j$  given that it is operating at time  $\tau_i$ , denoted by  $R(\tau_i, \tau_j)$ , is

$$R(\tau_i, \tau_j) = e^{-\lambda(\tau_j - \tau_i)} \quad (1.2)$$

$\tau_i < \tau_j$

## 2. Nonhomogeneous Poisson Process

The Nonhomogeneous Poisson Process, also called a nonstationary Poisson process, is a counting process similar to the HPP except that the rate function is not constant, but a function of system age,  $t$ , denoted by  $\lambda(t)$ . The  $x_i$ 's, i.e. the times between equipment failure, are not necessarily identically distributed. This mathematical structure can represent an increasing failure rate over time resulting from equipment wear. Pump data indicate such behavior.

Specifically, the counting process,  $\{N(t), t \geq 0\}$ , is said to be a Nonhomogeneous Poisson Process with age-dependent repairable failure rate  $\lambda(t)$  if:

1.  $N(t) = 0$ .
2.  $N(t)$  has independent increments.
3. The number of events, i.e. pump failures, in the interval  $(\tau_i, \tau_j)$  is Poisson distributed with mean  $m(\tau_i, \tau_j)$ ,

$$m(\tau_i, \tau_j) = \int_{\tau_i}^{\tau_j} \lambda(s) ds. \quad (1.3)$$

The probability of  $n$  failures in the time interval  $(\tau_i, \tau_j)$  is

$$Pr[N(\tau_j) - N(\tau_i) = n] = e^{-m(\tau_i, \tau_j)} \frac{(m(\tau_i, \tau_j))^n}{n!} \quad (1.4)$$

$\tau_i < \tau_j, n=0,1,\dots$

The reliability of the system at any time,  $t$ , depends on the age at which the most recent failure occurred. Suppose that a failure actually occurs at time  $t = \tau_j$ , where  $\tau_j$  is the age at which the  $j^{\text{th}}$  failure of the incumbent system (pump) occurs. Then the probability that the system will not fail for at least age  $\tau_1 + x$  units of time is

$$Pr[\tau_{j+1} - \tau_j \geq x | \tau_j = t] = e^{-m(\tau, \tau+x)} \quad (1.5)$$

## II. MODEL DEVELOPMENT

### A. DATA

The data used in this study consist of a sample set of sixty-one pumps and 140 failures over the observation period beginning in May 1987 and ending in December 1993. The final observation was either a failure and replacement prior to December 1993, or the pump was last observed still in operation. This sample set does not include all pumps in operation, but a subset for which there exists reliable dates for pump installation. Additionally, the pumps were carefully screened to ensure that all pump replacements resulted from non-repairable failures. The installation and failure times are rounded to the nearest month and henceforward all references to time and age will be in months. The time in service is adjusted to remove any inactivation period of two or more months in length from the pump's operating age. The pump itself has no runtime meter. Engineering logs monitor a pump's operation and would provide a more accurate measure of the actual operating time for each pump. However, it is not feasible to accumulate and analyze this information in a

reasonable time frame. Failure and replacement data originate from the Maintenance and Material Management (3-M) system which reports routine maintenance via OPNAV Form 4790/K, Casualty Reports (CASREP), the Submarine Maintenance, Engineering Planning and Procurement (SUBMEPP) system, and input directly from the Fleet.

Mr. Richard Youngk conducted a Failure Modes and Effects Analysis, FMEA, to identify the critical failure modes characteristic to the pump and characterize the required repair actions. Information on the types of failures, their frequency, the repairs requirements, and the trends for the different failure modes is essential elements in any analysis of equipment performance. This thesis focuses specifically on the occurrence of failures and the required repair action. Failures are not distinguished, but treated as one entity. The lifecycle cost in terms of material and labor is also examined. Appendix A is a copy of the actual data analyzed.

## **B. ASSUMPTIONS**

Several assumptions were made regarding the pump failure data. These assumptions are:

1. The submarines in the sample set have relatively similar operating cycles, i.e., pumps on different submarines will face the same operating conditions

and undergo roughly the same number of hours of operation for the same time period.

2. Every component failure causes equipment failure.
3. Failures are immediately evident.
4. Pumps are only repaired or replaced at failure and not in anticipation of failure. In actuality this is not always the case, but the data set was cleansed to ensure only replacements related to failures are counted.
5. Consecutive failures on an individual pump are independent.
6. A repair returns the pump to full operation, but does not necessarily restore it to a "good as new" condition.
7. Equipment repairs consume no appreciable time.
8. A replacement constitutes the installation of a new pump or a complete overhaul of the current pump.

The accuracy of these assumption and the accuracy of the data will be discussed further in Chapter IV.

### **C. ANALYZING THE DATA FOR TIME DEPENDENCY**

One of the initial steps in the analysis is to determine if the data are compatible with an increasing failure rate over time, i.e., with aging or wearout. Evaluating the time intervals between failures provides some insight into this matter. For a constant failure rate over time, the intervals between successive failures should be very similar; with all

interval times following an exponential distribution with the same mean. Care must be taken, however, to ensure all data has been taken into account. Invalid conclusions can be drawn, for example, by comparing the mean time to first failure with the mean interval time between the first and second failure if all pumps have not failed at least twice. Here the existing life of the pumps with only one failure is not accounted for in the mean for the second interval.

A better approach is to compare the mean interval time between successive failures for pumps with like failure numbers. Table 2.1 displays the mean and standard deviation (STD) for the interval times between failures for pumps with at least two failures, with at least three failures, and so on. In all cases the overall trend is a decreasing interval between failures over time, indicating an increasing failure rate. The correlation coefficient between successive failure times also provides evidence of an increasing failure rate. Table 2.1 also shows the correlation between the interarrival times of successive failures. The correlation between  $X_1$  and  $X_2$ , the interval times for failures one and two, and between  $X_2$  and  $X_3$  are not significant. The negative correlation between  $X_3$  and  $X_4$  and between  $X_4$  and  $X_5$ , however, indicates a decreasing interval time.

TABLE 2.1: COMPARISON OF INTERVAL TIMES BETWEEN FAILURES

FAILURES	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	SAMPLE NUMBER
2 (MEAN)	12.81	10.25					36
(STD)	7.97	8.03					
3	11.79	8.88	9.79				24
	8.05	6.54	8.78				
4	9.93	10.07	8.67	5.13			15
	6.82	7.01	7.90	4.77			
5	9.22	9.11	7.22	6.78	5.67		9
	4.76	6.33	4.61	5.29	4.57		
6	12.00	7.50	8.00	5.50	2.50	3.00	2
	1	3.5	5	3.5	1.5	1	
CORRELATION ( $X_1, X_{1+1}$ )	.029	.066	-.374	-.113	1.0	-	
SAMPLE NUMBER	36	24	15	9	2	-	

A more rigorous statistical test for distinguishing between a constant failure rate, i.e., a HPP, and a monotonic trend is the Laplace Test. The Laplace Test is based upon the HPP property that given  $n$  arrivals in time  $(0, F)$ , the unordered times of arrival, denoted by  $T_1, T_2, \dots, T_n$ , are the ordered statistics from an independent uniform random variable on  $(0, F)$ .



Therefore, the test statistic for the Laplace Test,  $U$ ,

$$U = \frac{\sum_{i=1}^n \left( \frac{T_i}{n} \right) - \frac{F}{2}}{F \sqrt{\frac{1}{12n}}} \quad (2.1)$$

has approximately a standard normal distribution. Bates (1955) showed this approximation was adequate at the 5% significance level for  $n \geq 4$ . [Ref.4]

A slight modification is required if the system is observed until a specific number of failures occur. In this reference case  $F = T_n$  and the modification is

$$U = \frac{\sum_{i=1}^{n-1} \left( \frac{T_i}{n-1} \right) - \frac{T_n}{2}}{T_n \sqrt{\frac{1}{12(n-1)}}} \quad (2.2)$$

Cox and Lewis (1966) showed that the Laplace test is optimal for the NHPP with rate

$$\lambda(t) = e^{\alpha + \beta t} \quad (2.3)$$

$-\infty < \alpha, \beta < \infty, t \geq 0$

in testing the hypothesis of a constant rate, i.e.,  $H_0:\beta=0$ , against the alternate hypothesis of a trend,  $H_a:\beta\neq 0$ .

The test can be further modified to examine a series of  $k$  independent systems with the same value of  $\beta$ . The new test statistic is, [Ref.5]

$$U = \frac{\sum_{j=1}^k \sum_{i=1}^{n_j} T_{ij} - \frac{1}{2} \sum_{j=1}^k n_j F_j}{\left( \frac{1}{12} \sum_{j=1}^k n_j F_j^2 \right)^{.5}} \quad (2.4)$$

This procedure was applied to 13 pumps meeting the criteria of four or more failures. Table 2.2 gives the sample set used for the test. The evaluation of two pumps, number 57 and 60, required the modification of Equation 2.2 since the observation period ended with a failure for both pumps.

The resulting value of the test statistic,  $U$ , was  $U = 2.016$  which resulted in a  $p$ -value of .04 for the two-tailed hypothesis test. Thus, the null hypothesis can be rejected in favor of a changing failure rate at the conventional 5% significance level. Further, the positive value of the test statistic indicates an increasing failure rate over time. This test included only 13 pumps of the original data set of 61 pumps. It is reasonable to assume, however, that this

TABLE 2.2: LAPLACE TEST FOR DATA TREND

LAPLACE TEST FOR A TREND IN THE FAILURE RATE				
$H_0: \beta = 0, H_a: \beta \neq 0$				
PUMP	FINAL OBSERVATIO N (MONTH)	STATUS C=RUNNING R=REPLACED	NUMBER OF FAILURES	SUM OF FAILURE TIMES
48	26	C	4	79
49	56	C	4	124
51	50	C	4	120
52	60	C	4	127
53	24	C	5	82
54	29	C	5	89
55	37	C	5	83
56	45	C	5	118
57	44	R	4*	67*
58	53	C	5	187
59	62	C	5	186
60	37	R	5*	127*
61	53	C	6	168
TEST STATISTIC VALUE : $U = 2.016$ P-VALUE = .043				
NOTES: * THE TOTAL NUMBER OF FAILURES IS MODIFIED FROM $n_j$ TO $n_{j-1}$ FOR PUMPS 57 AND 60 SINCE THE FINAL OBSERVATION WAS A REPLACEMENT. ** PUMP 50 IS NOT USED SINCE THE ADJUSTED NUMBER OF FAILURES IS 3.				

result is representative of the entire sample set. The Laplace Test was in fact performed on all pumps with at least one failure. This resulted in a value  $U = 1.912$  and  $P = .06$ . Although possibly not as accurate as for the case with  $n \geq 4$ , it does seem to indicate the overall trend is an increasing failure rate.

#### **D. CHOOSING A MODEL**

Given that the rate of failure occurrence seems to increase with pump age, the next step involves determining whether a mathematical model can be constructed to accurately simulate the occurrences of failures. The nonstationary trend of the data indicated that the times between successive repairable, i.e., non-fatal, failures were not identically distributed. A particular Non-homogeneous Poisson Process (NHPP) has been selected to model the occurrences of failure over the life of the pump. This approach was chosen based on the fact that

the NHPP assumption, besides being mathematically tractable, is a good representation of many systems in the real world because it is a consequence of the "minimal repair" assumption. "Minimal repair" implies that the repair involves the replacement of only a small fraction of a system's constituent parts. [Ref.6]

The following parametric failure rate,  $\lambda(t)$ , was used to model the increasing trend in failure over time:

$$\lambda(t) = e^{\alpha + \beta t} \quad (2.5)$$

$$-\infty < \alpha, \beta < \infty, t \geq 0$$

This form of failure rate was introduced by Cox and Lewis for the analysis of failure data for aircraft air-conditioning equipment in 1966.[Ref.7] Ascher and Feingold (1966) used this model to analyze the performance of submarine main propulsion diesel engines. This model has the advantage that since equation 2.5 is positive for all value of  $\alpha$  and  $\beta$ , no nonlinear restrictions are necessary for the estimators of these parameters.[Ref.8]

Unlike the Cox and Lewis situation, however, each failure may result in either a repair or a replacement. Therefore each failure is subject to a probability of being repairable, denoted by  $p(t_{i,j})$ , and a corresponding probability of being fatal and requiring pump replacement, denoted by  $q(t_{i,j})$  with  $q(t_{i,j}) = 1 - p(t_{i,j})$ . In fact, some repairs may be relatively easy while the platform is on a mission; others have greater operational impact in that they may require mission termination for repair to be made.

## E. MAXIMUM LIKELIHOOD ESTIMATION

The parameters  $\alpha$  and  $\beta$  have been estimated using the method of maximum likelihood. The maximum likelihood estimate, abbreviated as MLE, for  $\beta$  is found first, then  $\alpha$  is found. A numerical solution is necessary; there is no simple closed form solution for  $\beta$ .

Before deriving the MLE, it is necessary to define the following:

$k$  = index for the number of pumps in the sample set  
 $j$  = index for the individual pumps,  $j = 1, 2, \dots, k$   
 $i$  = index for the failure number for pump  $j$ ,  $i = 1, 2, \dots, n_j$   
 $n_j$  = the number of failures for pump  $j$   
 $f_j$  = time of the last observation for pump  $j$

\*Note: time is equivalent to pump age in this analysis\*

$t_{i,j}$  = time of failure  $i$  for pump  $j$   
 $p(t_{i,j})$  = probability that the failure occurring at time  $t_{i,j}$  is repairable.

$q(t_{i,j})$  = probability that the failure occurring at time  $t_{i,j}$  is not repairable and therefore requires pump replacement.  $q(t_{i,j}) = 1 - p(t_{i,j})$ .

$F_j$  = indicator variable indicating the status of the last observation

$F_j = 1$  for non-repairable failure

$F_j = 0$  for pump last observed operating

$\lambda(t_{i,j})$  = rate of occurrence of pump failures for pump age  $t_{i,j}$ .

The failure may or may not be repairable.

$\Lambda(t_{i,j})$  = the integrated value of the failure rate from pump installation to age  $t_{i,j}$ ; the expected number of failures (repairable) up to age  $t_{i,j}$

$\theta$  = vector of parameters,  $\theta = (\alpha, \beta)$

$L(\theta)$  = Combined Likelihood Function for pump failures

$L_c(\theta)$  = Conditional Likelihood Function for pump failures.

NOTE: If the last observation is a failure, then  $f_j = t_{n_j,j}$ .

Appendix B contains the complete derivation of the maximum likelihood estimators for  $\alpha$  and  $\beta$  and the following explanation will only deal with the final results of that derivation.

The combined likelihood function,  $L(\theta)$ , for all pump failures reduces to:

$$L(\theta) = e^{-\sum_{j=1}^k \lambda_j} \prod_{j=1}^k \prod_{i=1}^{n_j} \lambda_{ij} \prod_{j=1}^k \left( \frac{q(t_{n_j})}{p(t_{n_j})} \right)^{F_j} \prod_{j=1}^k \prod_{i=1}^{n_j} p(t_{ij}). \quad (2.6)$$

Substituting the  $\lambda(t)$  described by Equation 2.5 results in the following likelihood function:

$$L(\theta) = e^{\left[ \sum_{j=1}^k n_j \cdot \beta \sum_{i=1}^{n_j} t_{ij} - \sum_{j=1}^k \frac{e^{\beta}}{\beta} (e^{\beta t_{n_j}}) \right]} \prod_{j=1}^k \left( \frac{q(t_{n_j})}{p(t_{n_j})} \right)^{F_j} \prod_{j=1}^k \prod_{i=1}^{n_j} p(t_{ij}). \quad (2.7)$$

The conditional distribution of the observations given  $n_j$  events for pump  $j$  is formed by dividing the combined likelihood function by the marginal probability of pump  $j$  having  $n_j$  total failures. Taking the logarithm of this conditional distribution produces the conditional log likelihood function.

$$\log L_c(\theta) = \beta \sum_{j=1}^k \sum_{i=1}^{n_j} t_{ij} + \sum_{j=1}^k \log(n_j!) - \log \beta \sum_{j=1}^k n_j - \sum_{j=1}^k n_j \log(e^{\beta f_j} - 1) \quad (2.8)$$

The maximizing value for  $\beta$  is then obtained by maximizing  $\log [L_c(\theta)]$ . This can be accomplished by setting the derivative of the Conditional Log Likelihood Function, with respect to  $\beta$ , equal to zero and solving for the root,  $\beta$ .

$$\frac{d[\log L_c(\beta)]}{d\beta} = \sum_{j=1}^k \sum_{i=1}^{n_j} t_{ij} + \sum_{j=1}^k \frac{(n_j)}{\beta} - \sum_{j=1}^k \frac{(n_j)(f_j)}{(1-e^{-\beta f_j})} = 0. \quad (2.9)$$

Since no closed form solution for  $\beta$  exists, a numerical method is necessary to solve for the root. First,  $d[\log L_c(\theta)]/d\beta$  is plotted over a range of  $\beta$  for a visual approximation of the equation's root, Figure 2.2. This "best guess" is then used with Newton's Method to find a more precise value for the root. The resulting maximizing value of  $\beta$  is  $\hat{\beta} = 0.02258$ .



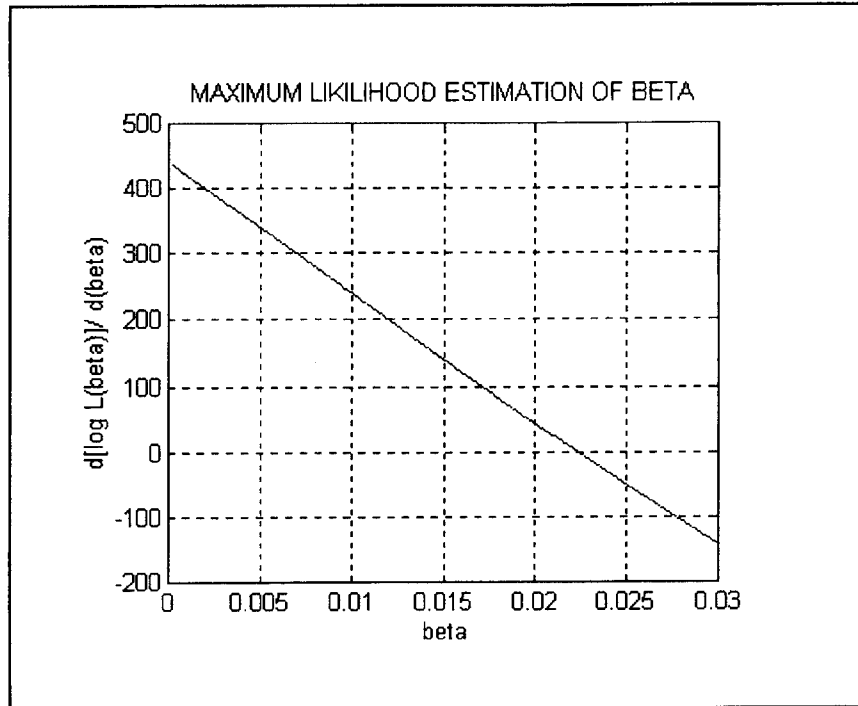


Figure 2.2: Visual Approximation of the MLE for  $\beta$

A similar process is used to find the value of  $\alpha$ . The conditional likelihood function, however, cannot be used to solve for  $\alpha$  as it was for  $\beta$  since the conditioning process removes  $\alpha$  as a parameter. Therefore,  $L(\theta)$  must be used to find the maximizing value of  $\alpha$ . Taking the logarithm of  $L(\theta)$ , differentiating with respect to  $\alpha$ , and setting the equation equal to zero results in the following expression:

$$\frac{d[\log L(\theta)]}{d\alpha} = \sum_{j=1}^k n_j - \frac{e^{\alpha}}{\beta} \sum_{j=1}^k (e^{\beta f_{j-1}}) = 0. \quad (2.10)$$

Solving the equation in terms of  $\alpha$  results in the following expression:

$$\hat{\alpha} = \ln \left[ \frac{\hat{\beta} \sum_{j=1}^k n_j}{\sum_{j=1}^k (e^{\beta f_{j-1}})} \right]. \quad (2.11)$$

Inserting the maximizing value of  $\hat{\beta} = 0.02258$  into Equation 2.11 produces the resulting maximizing value of  $\alpha$  is  $\hat{\alpha} = -3.189$ . Appendix C contains the *MathCad* 3.1 program of Newton's Method and the calculations used to determine the parameter values of  $\hat{\beta}$  and  $\hat{\alpha}$ .

The resulting maximum likelihood estimator for the failure rate of the pumps is:

$$\hat{\lambda}(t) = e^{-3.189 - 0.02258t}, \quad t \geq 0. \quad (2.12)$$

The confidence intervals for  $\hat{\alpha}$  and  $\hat{\beta}$  are obtained by inverting the observed information matrix to form an estimate of the variance-covariance matrix for  $\hat{\alpha}$  and  $\hat{\beta}$ . The unconditioned likelihood function, Equation 2.7, is used for calculating the confidence intervals since conditioning removes  $\alpha$  as a parameter. The resulting 95% Confidence Intervals are given in Table 2.3. Note that zero is not contained in the interval for  $\beta$ , giving further indication of an increasing failure rate. Appendix C contains the *Mathcad* 3.1 program used for the confidence interval calculations.

TABLE 2.3: CONFIDENCE INTERVAL FOR MLE OF  $\hat{\alpha}$  AND  $\hat{\beta}$

95% CONFIDENCE INTERVAL FOR $\hat{\alpha}$ AND $\hat{\beta}$			
PARAMETER	LOWER LIMIT	MLE	UPPER LIMIT
ALPHA	-3.50	-3.189	-2.87
BETA	0.012	0.02258	0.033

#### F. EVALUATION OF MODEL FIT

The use of any parametric model to quantify a system's behavior must also involve a goodness of fit analysis to determine the adequacy of the model. Two methods are used to assess the suitability of the afore mentioned NHPP as a model for the failure process. One method is graphical and the

other is a formal statistical procedure using Pearson's Chi-Squared Test.

The first method compares the an empirical failure rate taken over equidistant intervals to the model's failure rate for the corresponding times. The interval length selected was eight months. During each interval the total pump exposure was calculated as the sum of the number of pumps operating during each month of the interval. Similarly, the total number of failures for each interval was determined. The interval failure rate is simply a point estimate formed by dividing total failures by total exposure. The interval midpoint was used to calculate the model's failure rate,  $\lambda(t) = e^{\alpha + \beta t}$ . Table 2.4 contains the calculated rates and Figure 2.3 is the corresponding graph.

The model closely approximates the interval failure rate up to the last two intervals, i.e., intervals 49-56 and 57-64 months. This region indicates a decreasing failure rate. One should note, however, that the total exposure for these regions is relatively small compared to the other intervals. The small exposure, therefore, may not be representative of the overall trend. Further, one must recall the nature of the equipment being evaluated. The system is a cam-driven reciprocating pump; a mechanical system already noted to

TABLE 2.4: COMPARISON OF INTERVAL FAILURE AND THE  
MODEL FAILURE RATE

INTERVAL (MONTHS)	TOTAL EXPOSURE (MONTHS)	OBSERVED FAILURES	INTERVAL FAILURE RATE	MODEL FAILURE RATE
1-8	488	23	0.04713	0.04529
9-16	438	30	0.06849	0.05426
17-24	372	29	0.07796	0.06500
25-32	277	23	0.08303	0.07787
33-40	188	15	0.07979	0.09328
41-48	152	14	0.09211	0.11175
49-56	76	5	0.06579	0.13388
57-64	28	1	0.03571	0.16038

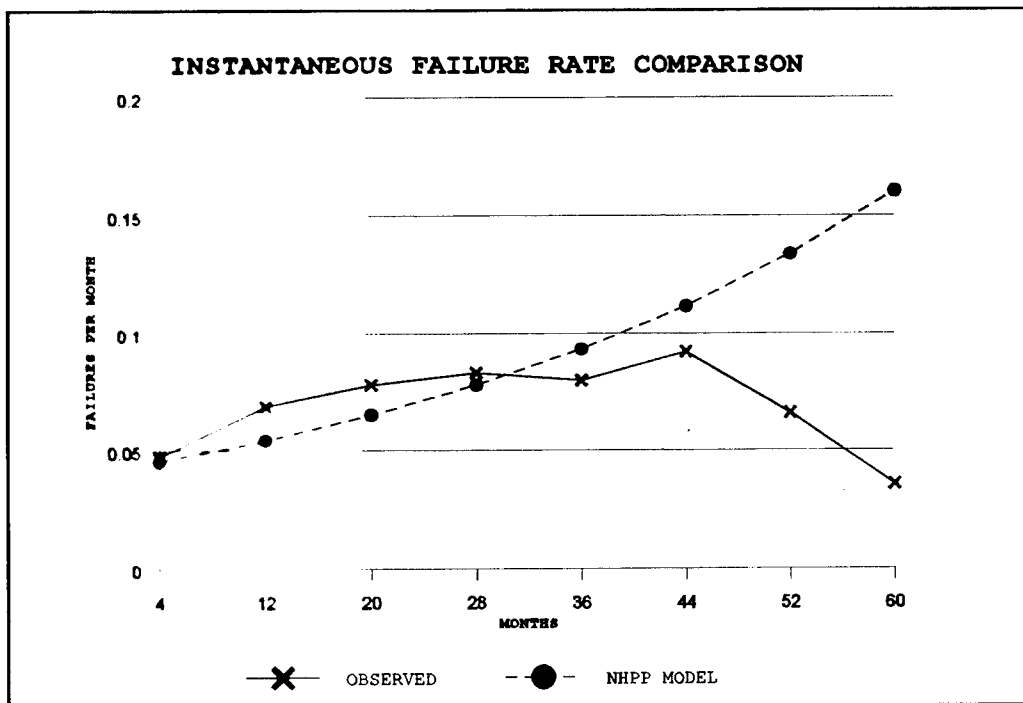


Figure 2.3: Observed Interval Failure Rate vs. the  
Model Failure Rate,  $\lambda(t) = e^{\alpha+\beta t}$

exhibit indications of wearout. Thus, the system is highly unlikely to experience any reliability improvement at this point in life. The system engineer at SMMSO independently reached a similar conclusion while doing a parallel study, i.e., that the observed decreasing failure rate late in life was a statistical fluctuation, resulting from sample size, and not truly typical of system performance. Of course it is possible that pumps on some vessels will actually improve with age, up to a point. This feature remains open for further investigation.

The Pearson Chi-Squared Test is a formal statistical test for goodness of fit. Here the expected numbers of failures under the NHPP assumption are compared with the observed numbers of failures. The same eight month intervals are used with the exception of the last two which are combined to provide at least five failures per interval. Table 2.5 contains the results. The chi-squared value is  $\chi^2 = 9.267$  with  $7-3=4$  degrees of freedom. This produces a p-value of  $0.05 < p < 0.1$ . Such a p-value is not ordinarily considered to indicate significant departure from the basic hypothesis, in this case the model.

Note the large contribution of the last interval. The same arguments as above can be made for this deviation. Even so,

one cannot reject the possibility that the data follows the NHPP model at a 5% significance level. Therefore, the model's representation of the data, although not perfect, appears acceptable. Note further that acceptance of the model is a conservative step in predicting future failure characteristics for the pump.

TABLE 2.5: CHI-SQUARED GOODNESS OF FIT TEST

INTERVAL (MONTHS)	TOTAL EXPOSURE (MONTHS)	OBSERVED FAILURES	EXPECTED FAILURES	CHI SQUARED VALUE
1-8	488	23	22.1	0.0341
9-16	438	30	23.8	1.6172
17-24	372	29	24.2	0.9468
25-32	277	23	21.6	0.0910
33-40	188	15	17.6	0.3735
41-48	152	14	17.0	0.5324
49-64	104	6	15.3	5.6717
CHI SQUARED VALUE: $\chi^2 = 9.267$ with $7-31 = 4$ DF P-VALUE: $.05 < p < .1$				

### III. DEVELOPING A REPLACEMENT POLICY

#### A. BENEFITS OF STOCHASTIC MODELING

The benefit of developing a stochastic model of the cam-driven reciprocating pump's failure characteristics is that pump reliability is now described by a simple mathematical expression that describes likely future behavior. One might argue that empirical data would provide a better representation of actual pump performance. This is true for systems with an extensive data base on lifecycle performance. Such an extensive data base, however, does not exist for the cam-driven reciprocating pump. In fact, very little information exists for pumps over 60 months age. The lack of such data demonstrates the need for a probabilistic model to predict future behavior. The mathematical model allows easy calculation of the expected number of pump failures, and its availability for different pump ages if downtimes are also modeled. It also expedites determination of an optimal replacement interval based on costs.

This chapter will discuss the merits of time-based replacement and evaluate the applicability and effectiveness



of a time-based replacement policy for the cam-driven reciprocating pump. The model and results developed in Chapter II provide the basis for the evaluation.

#### **B. CURRENT MAINTENANCE POLICY**

The cam-driven reciprocating pump is currently on a time-based replacement schedule. The schedule varies, however, between platform types, specifically between Trident and non-Trident submarines. The same pump is used in both platforms, although slight differences in the systems exist. Current replacement intervals for Trident and non-Trident assets are approximately 36 and 60 months, respectively.

The time-based replacement policy involves the replacement of the cam-driven reciprocating pump at a predetermined age regardless of the current material condition of the pump. The value of this time-based replacement in maintenance planning is obvious. A planned maintenance evolution allows for the scheduling and prepositioning of parts, personnel, and support facilities to minimize system downtime and thus tends to minimize total system (submarine) nonavailability. It also precludes possible extensive and expensive failures that may occur at later ages.

This reasoning has led to the frequent and possibly excessive use of time-based maintenance by the Navy.

The Navy uses time-based maintenance for components, equipment and systems ranging in complexity from oil filters to propulsion gas turbines. Most of the maintenance action in Class Maintenance Plans are based on engineering time based periodicities. RCM [Reliability Centered Maintenance] requires that these intervals be adjusted based on equipment performance and failure rate. Most time-based overhauls/refurbishments/replacements are also expensive. CBM [Condition Based Maintenance], applied where appropriate, will greatly reduce the number of time-base repairs and overhauls conducted. The key to successful implementation of CBM is application of the proper level of monitoring, evaluation and trending for each piece of equipment. [Ref.9]

The challenge, therefore, is to evaluate the appropriateness and effectiveness of the current time-based maintenance practices and to optimize the interval used. The following issues must be addressed for the cam-driven reciprocating pump:

1. Is this pump an appropriate candidate for time-based replacement?
2. If time-based replacement is warranted, what is the optimal replacement interval to minimize cost?

### **C. CRITERIA FOR TIME-BASED REPLACEMENT**

The goal of time-based replacement is to improve both the current and long-run operating state of the system through preplanned maintenance actions. The meaning of "operating

state" is dependent upon the nature and mission of the system in question and upon the objectives of the policy makers. Thus, "operating state" refers to operating cost, system reliability, maintainability, and so on.

MIL-STD-2173(AS) provides the following guidance on the applicability of time-based tasks, also referred to as hard time tasks. Here the time-based task is considered to be of two types, scheduled rework or scheduled discard. A reworking task is analyzed if reworking promises to restore the item to an acceptable level of failure resistance; otherwise, the discard task is analyzed.

The applicability criteria for time-based (hard time) tasks are as follows:

1. The item must be capable of having an acceptable level of failure resistance after being restored (for rework task).
2. The item must exhibit wearout characteristics, which are identified by an increase in the conditional probability of failure with increasing usage(age). This property can lead to establishment of a *wearout age* (for rework tasks) or a *life-limit* (for discard tasks).
3. A large percent of the items must survive to the *wearout age* or *life-limit*.
4. A safe *life-limit* for an item must be established at an age below which relatively few failures are expected to occur. [Ref.10]

MIL-STD-2173(AS) points out two key elements for time-based replacement, namely an increasing failure rate, i.e., (2) above; and a large rate of survival to the wearout age or life-limit, (3) above. An increase in the occurrence of repairable failures with system age will often tend to result in a corresponding increase in maintenance costs. A decrease in system reliability and availability will also occur. From these two viewpoints, optimal replacement intervals should exist to minimize costs and/or maintain the system above certain minimum reliability and availability requirements.

A large percent of items must survive to the point of wearout, i.e., point of increasing failure rate, or life-limit, to make planned replacements an effective maintenance tool. Since a scheduled replacement is deemed to be more desirable than an unscheduled one, the opportunity to make replacements should be utilized fully. Of course, the resources associated with the logistics of planning a maintenance evolution may be wasted if a large percentage of the replacements are premature and occur prior to the planned interval.

Several additional factors must also be considered in determining the applicability of time-based replacement. Even a system with a constant failure rate may warrant time-based

replacement if an increase in operating and maintenance costs occurs as the item ages, or if repairs at sea are less easily made. Failures may occur no more frequently with system age, but the nature and type of repairable failures may result in an increase in the cost of parts and labor as the system ages. Another consideration in a time-based replacement scheme is the scope of work required to replace the component and the maintenance requirements of neighboring systems. Some replacements may require extensive interference removal, elaborate pre-established plant conditions, and extensive post-installation testing. In such cases, common sense dictates combining maintenance actions requiring the same or similar conditions. Such scheduling may not coincide with the optimal interval to minimize the operating cost for every item or subsystem, but even if some compromise is required the overall savings could be substantial. This thesis does not address the problem of coincident replacement of sets of different subsystems that have age-dependent failure properties.

#### **D. APPLICABILITY OF THE CAM-DRIVEN RECIPROCATING PUMP**

The cam-driven reciprocating pump is a candidate for time-based replacement. The data analysis of Chapter II suggests

an increasing occurrence of repairable failures as the pump ages, thus meeting criteria (b) for applicability. As stated earlier, the cam-driven reciprocating pump is currently on a time-based replacement schedule of 36 and 60 months for Trident and non-Trident assets, respectively. The remainder of this chapter will examine available data for evidence that the data supports the current replacement intervals.

#### **E. DERIVING A COST MINIMIZING FUNCTION**

As stated earlier, a stochastic model of the pump's failure characteristics allows the use of mathematical methodology to predict and quantify future behavior. One such tool is the Renewal Reward Process.

Recall that the Nonhomogeneous Poisson Process is used to model individual pump failure times. The NHPP is not a renewal process, but since all new pumps are assumed to be similar, the number of pump replacements to occur in  $(0, t)$  does constitute a renewal process. Let  $M(t)$  represent the number of pump replacements occurring in a system up to and including time  $t$ , and let  $L_n$ ,  $n \geq 0$ , represent the interval time between pump replacements. The cost associated with each renewal is denoted by  $R_n$ . It is assumed that  $\{L_n\}$  and  $\{R_n\}$  are sequences of identically distributed random variables; a

generic  $L_n$ , or  $R_n$ , is denoted by  $L$ , or  $R$ . Then let  $R(t)$  be the sum of all system costs incurred by time  $t$ , so

$$R(t) = \sum_{n=0}^{M(t)} R_n \quad (3.1)$$

Further, denote the expected values for  $R_n$  and  $L_n$  as follows:  $E[R] = E[R_n]$  and  $E[L] = E[L_n]$ . Then the following proposition holds:

**Proposition 3.1.** [Ref.11]

If  $E[R] < \infty$  and  $E[L] < \infty$ , then

$$\lim_{t \rightarrow \infty} \frac{R(t)}{t} = \frac{E[R]}{E[L]}$$

Let a cycle denote an individual pump's life. The proposition states that the long-run average cost equals the average cycle cost, i.e., repair and replacement costs, divided by the average cycle length, i.e., pump life. More precisely, the long-run average system cost equals

$$\frac{E[\text{cost incurred during a pump's life}]}{E[\text{pump life}]} \quad (3.2)$$

Pump life is a function of the replacement interval and the pump's mortality. Thus, a replacement interval for minimizing costs can be found by minimizing the long-run average cost, as given by Equation 3.2, over different replacement intervals.

### 1. Expected Pump Life

Let  $X$  be a random variable representing the age of the pump at replacement and let  $T$  be the designated maximum replacement age, in months. Further, define a cycle to be the interval from pump installation to pump replacement as defined by actual replacement or complete overhaul. Then the cycle length, denoted by  $L$ , equals  $X$  if the pump fails and requires replacement prior to the  $T$ ; otherwise,  $L$  equals  $T$  if the pump life is at least as long as the scheduled replacement interval. Thus the cycle length can be summarized by

$$L = \begin{cases} X & \text{if } 0 \leq X < T \\ T & \text{if } T \leq X \end{cases} \quad (3.3)$$

It is assumed that the probability a failure is repairable is constant, i.e., independent of pump age and the failure number; that is, a failure is repairable with



constant probability  $p$ . The data have 140 failures, of which 126 were repairable. This results in a point estimate for  $p$  of  $(126/140) = 0.9$ . The approximate normal 95% Confidence Interval for  $p$  is  $(0.85, 0.95)$ . Likewise, define the probability that a failure is not repairable as  $q$ ,  $q=1-p$ . Brown and Proschan used a similar assumption in their imperfect-repair model. The Brown and Proschan model assumes that the mode of repair was based solely on external conditions and not on the condition of the system at failure. [Ref.12]

Table 3.1 is a contingency table showing the distribution of failures, conditioned on failure number. The assumption is that the proportion of non-repairable failures,  $q$ , remains constant over the number of failures. Let  $n_i$  denote the number of pumps having at least  $i$  failures,  $i = 1, 2, \dots, 6$ . Further, define  $O_{i,j}$  as the number of observed replacements and  $O_{i,1}$  as the number of repairs for pumps with at least  $i$  failures. Define the total number of replacements and repairs as  $R_0$ , and  $R_1$ , respectively. The hypothesis of a constant proportion of non-repairable failures is tested by comparing the observed and expected values under the null hypothesis. Note that the observations for failures 5 and 6 are combined due to the small sample size. The test results in a value of

$\chi^2 = 1.13$  with 4 degrees of freedom for a p-value of  $0.8 < p < 0.9$ . The hypothesis of a constant is accepted at the 5% significance level. The small data set does not provide for the most accurate test, but it does give some indication of the goodness of fit. So the assumption of a constant probability for a failure being repairable is not unrealistic.

TABLE 3.1: TEST FOR CONSTANT REPLACEMENT PROPORTION

FAILURE NUMBER						
STATUS	1	2	3	4	5+6	TOTALS
REPLACEMENTS	5	3	2	2	2	$R_0=14$
REPAIRS	49	33	22	13	9	$R_1=126$
$\eta_i$	54	36	24	15	11	140
CHI-SQUARED VALUE: $\chi^2 = 1.13$ , DF = 4 P-VALUE: $0.8 < p < 0.9$  $\chi^2 = \sum_{i=1}^5 \sum_{j=0}^1 \left( (O_{ij} - \frac{\eta_i R_j}{140})^2 / \frac{\eta_i R_j}{140} \right)$						

Now define  $h(x)$  as the probability density function for a pump's life,  $\mathbf{x}$ . Then  $h(x)$  is

$$h(x) = \sum_{n=0}^{\infty} e^{-\Lambda(x)} \frac{(\Lambda(x))^n}{n!} p^n \lambda(x) q = e^{-q\Lambda(x)} q \lambda(x). \quad (3.4)$$

Similarly, the probability that the pump does not fail before the scheduled replacement,  $P(X \geq T)$ , is

$$P(X \geq T) = \sum_{n=0}^{\infty} e^{-\Lambda(T)} \frac{(\Lambda(T))^n}{n!} p^n = e^{-q\Lambda(T)}. \quad (3.5)$$

Using the above information it is possible to now derive an expression for the expected or mean life of a pump with a designated replacement interval of time  $T$ . In view of (3.3), the expected life, denoted by  $E[L]$ , is

$$E[L] = \int_0^T x h(x) dx + \int_T^{\infty} T h(x) dx. \quad (3.6)$$

Inserting Equation 3.4 and further simplification results in the following expression for mean pump life:

$$E[L] = \int_0^T x e^{-\Lambda(x)q} \lambda(x) q dx + T e^{-\Lambda(T)q}. \quad (3.7)$$

The above general formula can be specialized to account for any parametric form for  $\lambda(t)$ . Often the formula must be numerically evaluated; closed-form expressions may not exist.

## 2. Expected Cycle Costs

The cost incurred over the life of a pump consists of two components; the cost of a new pump; and repair costs incurred until the next replacement. Thus the life cycle cost of the pump is influenced by the number of repairable failures occurring over the pump's life. The expected life-cycle cost can be represented as

$$E[\text{life-cycle cost}] = \text{COST}_{\text{new}} + \text{COST}_{\text{repair}} \cdot E[\text{number of repairs}]$$

where both  $\text{COST}_{\text{new}}$  and  $\text{COST}_{\text{repair}}$  are expected or mean values.

As stated earlier, the pump's life can terminate in one of two ways; the pump may have a non-repairable failure prior to scheduled replacement, or the pump will be replaced per the schedule at age  $T$ . Let  $\mathbf{N}(t)$  represent the number of repairable failures occurring in the system up to and including time  $t$ . Likewise let  $E[\mathbf{N}]$  represent the expected number of repairable failures occurring during pump life.

Then

$$E[\mathbf{N}] = E[\mathbf{N}(X)|X < T] \cdot P(X < T) + E[\mathbf{N}(T)|X \geq T] \cdot P(X \geq T). \quad (3.8)$$

The analysis will examine each case individually.

**a. Case  $X < T$**

The expected number of failures given that the pump does not live to the scheduled replacement age  $T$  is

$$E[N(X)|X < T] \cdot P(X < T) = \sum_{n=0}^{\infty} n \int_0^T e^{-\Lambda(x)} \frac{(\Lambda(x)p)^n}{n!} \lambda(x) q dx. \quad (3.9)$$

This simplifies to

$$E[N(X)|X < T] \cdot P(X < T) = qp \int_0^T \Lambda(x) e^{-\Lambda(x)q} \lambda(x) dx. \quad (3.10)$$

Integration by parts results in the following closed form expression:

$$E[N(X)|X < T] \cdot P(X < T) = \frac{p}{q} - p\Lambda(T)e^{-\Lambda(T)q} - \frac{p}{q}e^{-\Lambda(T)q}. \quad (3.11)$$

**b. Case  $X \geq T$**

The expected number of repairs given the pump survives to scheduled replacement is

$$E[N(T)|X \geq T] \cdot P(X \geq T) = \sum_{n=0}^{\infty} n e^{-\Lambda(T)} \frac{(\Lambda(T)p)^n}{n!}. \quad (3.12)$$

This expression reduces to

$$E[N(T)|X \geq T] \cdot P(X \geq T) = \Lambda(T)p e^{-\Lambda(T)p}. \quad (3.13)$$

Finally combining Equations 3.11 and 3.13 the expression for the expected number of repairs over the pump's life is

$$E[N] = \frac{p}{q} - p\Lambda(T)e^{-\Lambda(T)p} - \frac{p}{q}e^{-\Lambda(T)p} + \Lambda(T)p e^{-\Lambda(T)p}. \quad (3.14)$$

The mathematical expression for the expected cost incurred during the pump's life is

$$E[R] = \text{COST}_{\text{new}} + \text{COST}_{\text{repair}} \left[ \frac{p}{q} - p\Lambda(T)e^{-\Lambda(T)p} - \frac{p}{q}e^{-\Lambda(T)p} + \Lambda(T)p e^{-\Lambda(T)p} \right] \quad (3.15)$$

As with Equation 3.8, the above general formula can be specialized to account for any parametric form for  $\Lambda(t)$ .

### 3. Long-run Average System Cost

The long-run average system cost can now be expressed in terms of Equations 3.7 and 3.15. Let  $z(T)$  denote the long-run cost average for the replacement interval of length  $T$ . Then

$$z(T) = \frac{COST_{new} + COST_{repair} \left( \frac{p}{q} - p\Lambda(T)e^{-\Lambda(T)q} - \frac{p}{q}e^{-\Lambda(T)q} + \Lambda(T)pe^{-\Lambda(T)q} \right)}{\int_0^T x e^{-\Lambda(x)q} \lambda(x) q dx + T e^{-\Lambda(T)q}} \quad (3.16)$$

Inserting the following parametric expressions for  $\Lambda(t)$  and  $\lambda(t)$ :

$$\Lambda(x) = \frac{e^{\alpha}}{\beta} (e^{\beta x} - 1) \quad \lambda(x) = e^{\alpha - \beta x},$$

and  $p = 0.9$ , the optimal replacement interval is found by minimizing  $z(T)$ , displayed in Equation 3.16, over  $T$ .

### F. DETERMINING THE OPTIMAL PUMP REPLACEMENT INTERVAL

The first step is to determine representative values for the average replacement and repair costs. Here the replacement cost is simply the cost of a new pump which is

**DISTRIBUTION OF FAILURE TYPE**  
(OVER PUMP LIFE)

Y-axis: FAILURE TYPE (0 to 14)  
X-axis: PUMP AGE (MONTHS) (0 to 70)

Pump Age (Months)	Failure Type
2	5
3	4
4	4
5	4
5	9
6	3
6	6
6	9
7	9
8	9
9	7
9	11
9	13
10	4
10	5
10	8
10	12
10	13
11	1
11	13
12	3
12	4
12	8
12	9
13	1
13	7
13	11
14	9
15	4
15	6
15	8
15	9
15	11
16	1
16	5
16	6
16	8
16	9
16	11
17	4
17	5
17	6
17	8
17	9
17	11
18	4
18	5
18	6
18	8
18	9
18	11
19	4
19	5
19	6
19	8
19	9
19	11
20	4
20	5
20	6
20	8
20	9
20	11
21	4
21	5
21	6
21	8
21	9
21	11
22	4
22	5
22	6
22	8
22	9
22	11
23	3
23	4
23	5
23	6
23	8
23	9
23	11
24	3
24	4
24	5
24	6
24	8
24	9
24	11
25	2
25	3
25	4
25	5
25	6
25	8
25	9
25	11
26	1
26	2
26	3
26	4
26	5
26	6
26	8
26	9
26	11
27	1
27	2
27	3
27	4
27	5
27	6
27	8
27	9
27	11
28	1
28	2
28	3
28	4
28	5
28	6
28	8
28	9
28	11
29	3
29	4
29	5
29	6
29	8
29	9
29	11
30	3
30	4
30	5
30	6
30	8
30	9
30	11
31	3
31	4
31	5
31	6
31	8
31	9
31	11
32	5
32	6
32	8
32	9
32	11
33	5
33	6
33	8
33	9
33	11
34	5
34	6
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34	9
34	11
35	5
35	6
35	8
35	9
35	11
36	4
36	5
36	6
36	8
36	9
36	11
37	4
37	5
37	6
37	8
37	9
37	11
38	4
38	5
38	6
38	8
38	9
38	11
39	4
39	5
39	6
39	8
39	9
39	11
40	4
40	5
40	6
40	8
40	9
40	11
41	4
41	5
41	6
41	8
41	9
41	11
42	4
42	5
42	6
42	8
42	9
42	11
43	4
43	5
43	6
43	8
43	9
43	11
44	4
44	

46



This leads to a more useful representation of repairs cost: a moving average over pump age. Using information on material costs for repairs from the 3-M System, a rough estimate of repair costs for the different repair types was found. The 3-M data is sketchy, however, and information was not available for all repair types.

The construction of the moving average involved ordering the failure times and assigning an average repair cost to each based on the repair type. The moving average consisted of the average repair costs for ten consecutive failures. Figure 3.2 show this average cost as a function of time. Due to the limited information on repair costs, no attempt was made to further quantify the relationship of repair costs with age. No obvious trend is evident from the moving average, so for a conservative estimate of repair costs, the final average occurring at month 61, \$4900, will be used as a basis for replacement interval determination. The repair cost estimate deals only with material costs. Other factors not captured in this cost estimate include labor costs and overhead costs. The 3-M System contains limited data on labor manhours for repairs, but again this information is sketchy. Similarly to the material cost projections, a ten point moving average was constructed to assess the changing maintenance

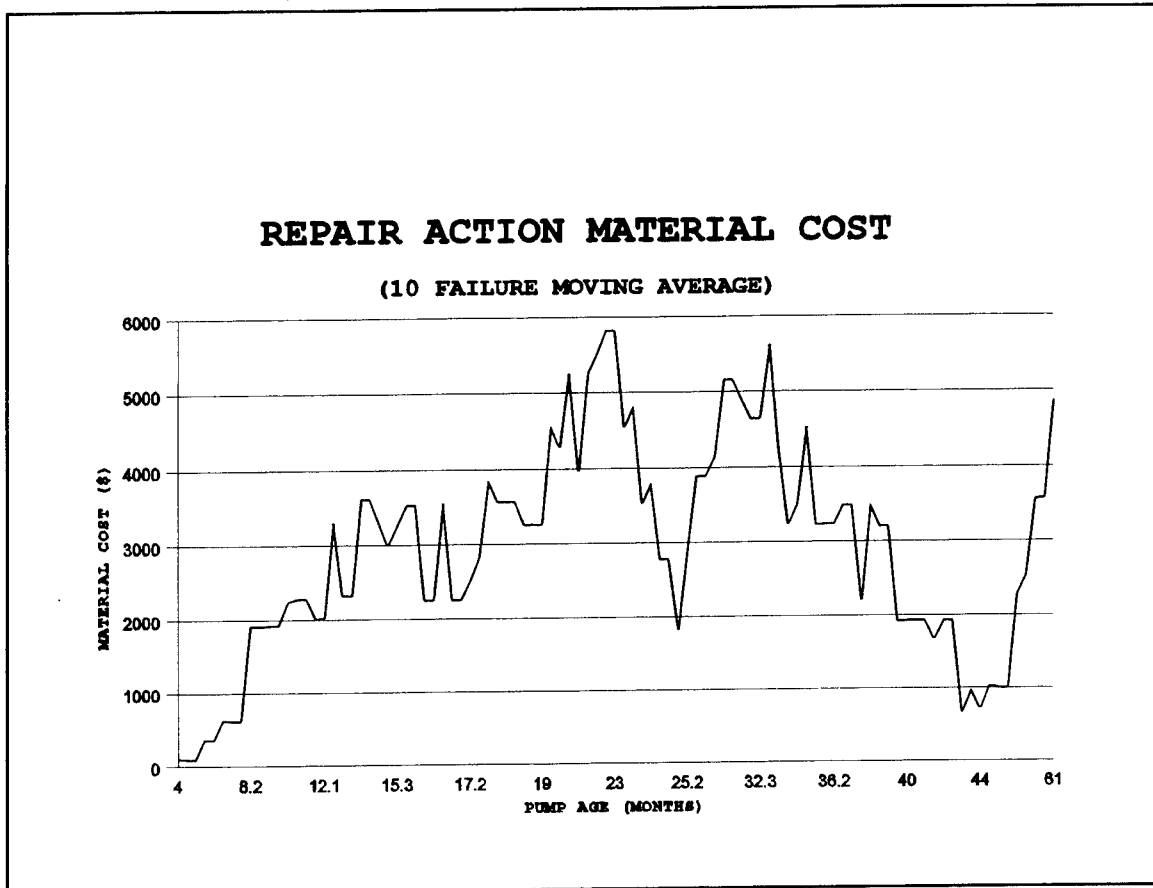


Figure 3.2: Repair Cost Moving Average (10 pt. Avg)

requirements as the pump ages. No attempt was made to assign a dollar figure to these labor projections due to the complexity of the task and the lack of quality data. Future research, however, should be directed at developing such a cost relationship to be combined with material costs for a comprehensive evaluation of trends in maintenance costs.

Figure 3.3 show the resulting graph of the moving average for expended manhours for repairs as the pump ages. The graph shows a possible increase in labor requirements with pump age.

As stated earlier, no attempt is made to incorporate this data into the cost estimation.

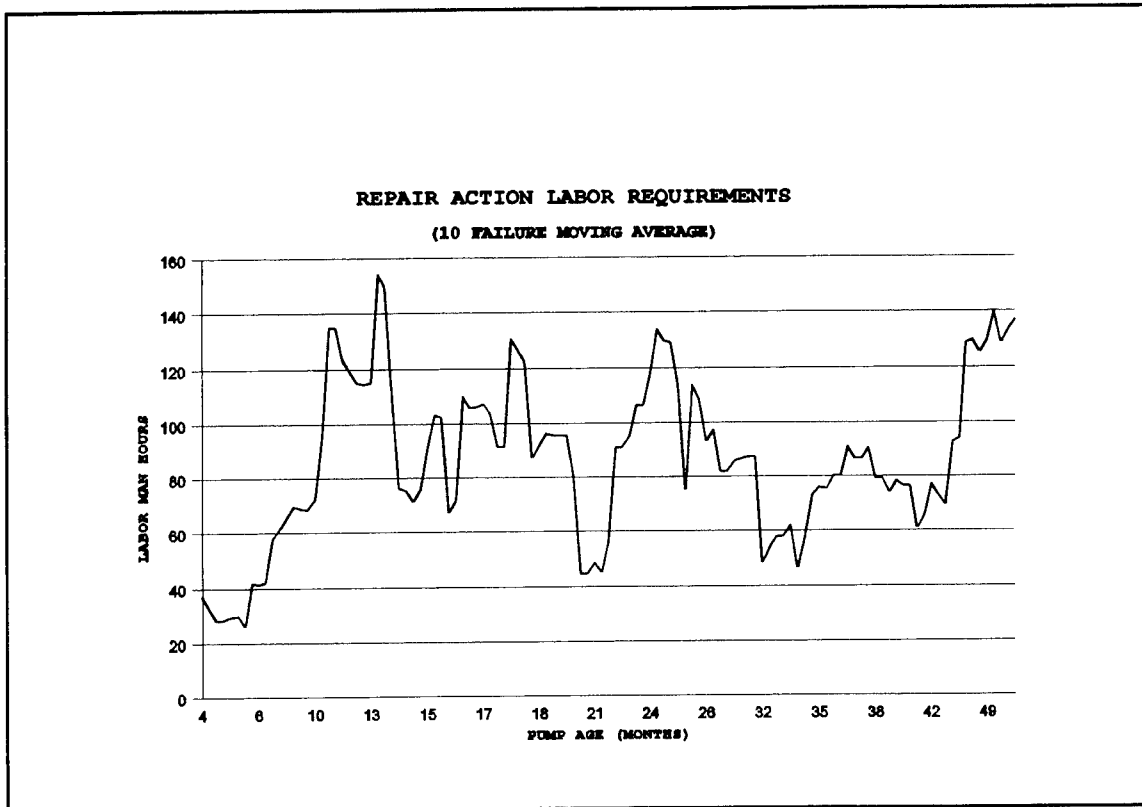


Figure 3.3: Moving Average Repair Labor Requirements  
(10 pt. AVG)

Determining the replacement interval is now simply a matter of minimizing  $z(T)$  over  $T$ . A *Mathcad 3.1* program was used to evaluate and graph  $z(T)$  over a range of  $T$ . The optimal replacement interval is the minimum point on the graph. The optimal value of  $T$  is rounded to the nearest month. Table 3.2 contains the results for the MLE of  $\alpha$  and  $\beta$  as well as for their 95% Confidence Bounds. Figure 3.4 is

a graph of  $z(T)$  for the MLE of  $\alpha$  and  $\beta$ . Calculations are made for both the final moving average value of \$4900 and the overall average repair cost value of \$3000. Calculations are likewise made using the 95% Confidence bound derived in Chapter II.

TABLE 3.2: OPTIMAL REPLACEMENT INTERVAL

$C_{\text{New}} = \$111,000$ $p = 0.9$		REPLACEMENT INTERVAL (MONTHS)	
$\alpha$	$\beta$	$C_{\text{Repair}} = \$4900$	$C_{\text{Repair}} = \$3000$
<b>-3.189</b>	<b>.0226</b>	<b>111</b>	<b>128</b>
-3.50	.0226	118	135
-2.87	.0226	104	121
-3.189	.012	183	214
-3.50	.012	195	227
-2.87	.012	171	202
-3.189	.033	82	94
-3.50	.033	88	99
-2.87	.033	77	88

The results indicate an optimal replacement interval of 111 and 128 months for the average repair costs of \$4900 and \$3000, respectively. These figures as well as the calculated interval using the confidence bounds are well above the current replacement intervals of 36 and 60 months for Trident and non-Trident platforms, respectively. The failure and cost

data, therefore, may not justify the current Trident and non-Trident platforms, respectively. The failure and cost data, therefore, may not justify the current replacement interval on the basis of minimizing maintenance cost alone.

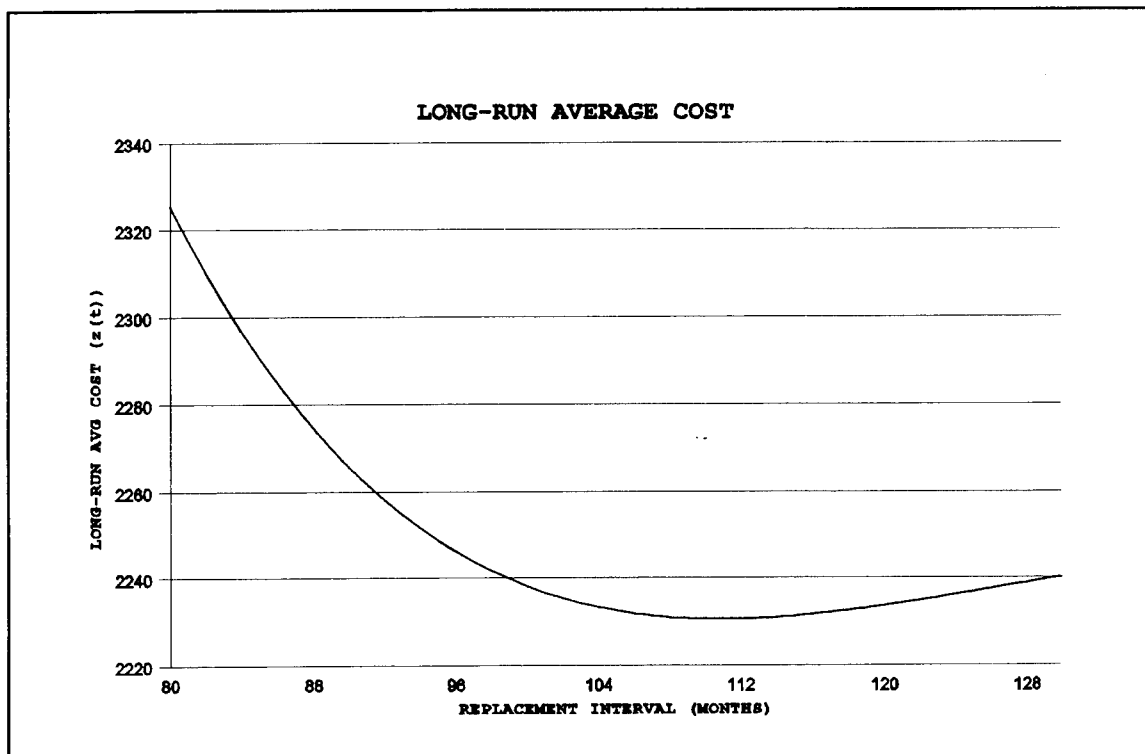


Figure 3.4: Long-Run Average Cost

Recall, however, that only material costs are used for the repair cost estimate. Other costs associated with repair include the cost of labor and shipyard facilities. Even so, an average repair cost of approximately \$32,000 would be required for an optimal replacement interval of 60 months

using the MLE parameters. The lowest average repair cost required for a 60 month interval is \$11,000 and occurs at the bounds of  $\alpha = -2.87$  and  $\beta = 0.033$ . Other considerations not accounted for here, however, may be included in the current replacement policy.

In developing a replacement policy for the cam-driven reciprocating pump, one must not lose sight that the pump is installed in a warfighting ship, namely a submarine. Cost is not and should not be the lone factor in establishing maintenance policy. System reliability and its effect on overall mission accomplishment must be a strong consideration in any decision. To aid decision makers, the following estimates of pump performance are calculated using the model: expected number of failures for a specific pump age, the expected failure times, and pump reliability for a specified mission duration.

Since a NHPP with rate  $\lambda(t)$  is used to model the failure characteristics of the pump the mean value function, Equation 1.3, is used to calculate the expected number of failures over pump age. Figure 3.5 shows the expected total number of failures as the pump ages if the pump is never replaced. Recall that each failure is assumed to have a probability of

$q = 0.10$  that the failure is non-repairable. The sequence of failures, thus constitute a geometric distribution. The mean number of failures before a required replacement is  $(1/q)$  or 10 failures, i.e., a replacement will be required for the tenth failure.

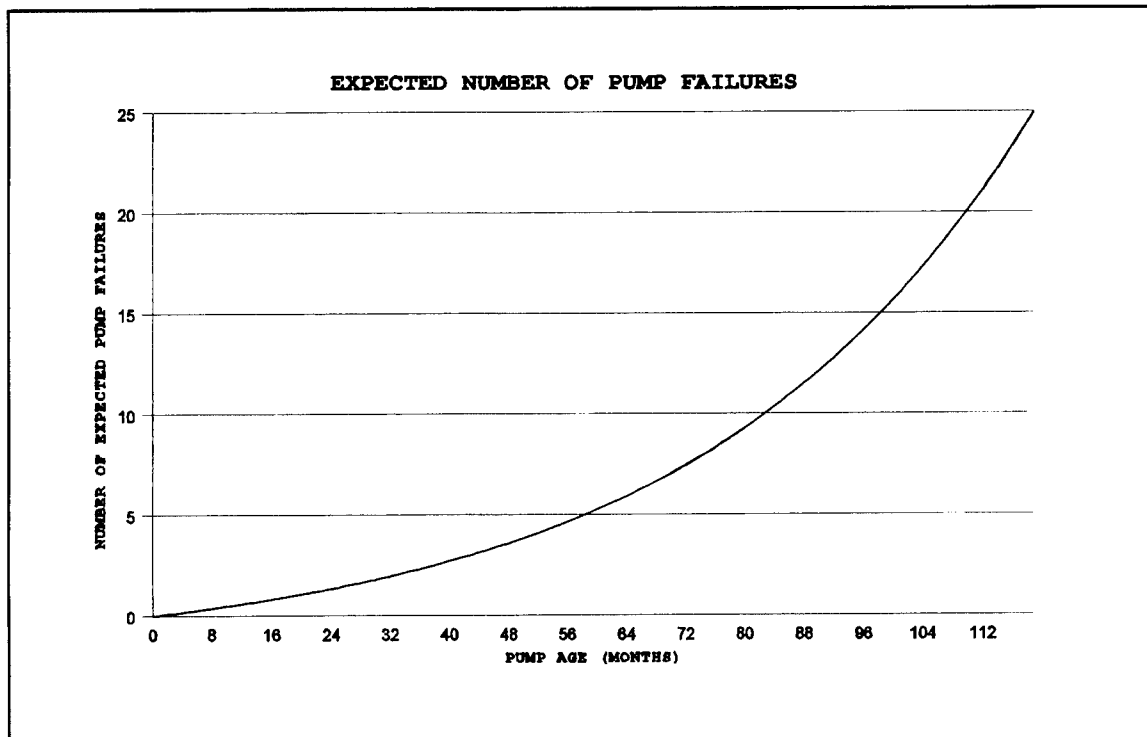


Figure 3.5: Expected Number of Pump Failures

The average times to failure can be calculated using a simple simulation program. Since the NHPP possesses a continuous mean value function  $\Lambda(t)$ , Çinlar (1975) discussed the following recursive algorithm for generating a sequence of arrival times,  $t_1, t_2, \dots, t_n$ :

1. Calculate the expectation function  $\Lambda(t)$  and its inverse  $\Lambda^{-1}(t)$ :

$$\Lambda(t) = \int_0^t \lambda(x) dx = \int_0^t e^{\alpha + \beta x} dx$$

$$X = \Lambda(t) = \frac{e^{\alpha}}{\beta} (e^{\beta t} - 1)$$

$$\Lambda^{-1}(X) = \frac{1}{\beta} \ln\left(\frac{X\beta}{e^{\alpha}} + 1\right)$$

2. Generate a random variable  $U \sim U(0,1)$ .
3. Set  $t'_i = t'_{i-1} - \ln(U)$ .
4.  $t_i = \Lambda^{-1}[t'_i]$ . [Ref.13]

Table 3.3 contains the mean pump age at failure using 500 replications of the above algorithm for simulated failure times.

The decision to replace or repair a pump may be guided by certain minimum requirements for mission reliability. Recall from Chapter I that the reliability of the system at any time  $t$ , depends on the age at which the most recent failure occurred. Equation 1.5 gives the expression for the reliability of the system at that time. Figure 3.6 is a graph of the probability that a pump having a repairable



TABLE 3.3: SIMULATED PUMP FAILURE TIMES

FAILURE NUMBER	MEAN PUMP AGE (MONTHS)	INTERVAL BETWEEN FAILURES	STANDARD ERROR OF MEAN
1	16.6	-	.602
2	29.2	12.6	.672
3	39.7	10.5	.691
4	48	8.3	.681
5	55.4	7.4	.659
6	61.8	6.4	.634
7	67.5	5.7	.624
8	72.4	4.9	.589
9	76.8	4.4	.555
10	81.1	4.3	.539
11	84.7	3.6	.517
12	88.1	3.4	.501
13	91.1	3	.489
14	94.1	3	.479

failure at age  $t$  will successfully complete a mission of three months duration without experiencing a failure. Figure 3.7 shows the expected number of failures that will occur over a 3 month mission given the pump is age  $t$  at mission commencement. This is computed using Equation 1.3 for pump ages  $(t, t+3)$ .

Both Figures 3.6 and 3.7 indicate that the replacement interval of 111 months, based entirely on material costs, may not be desirable due to the poor performance of the pump in terms of mission survivability, and expected pump failures during a mission. The current replacement interval of 60 months may be acceptable based upon minimum mission reliability standards. No attempt here is made to define those standards. The replacement interval of 36 months is probably premature and should be extended at least to 60 months to coincide with the policy for non-Trident submarines.

Table 3.4 provides the estimated mission survivability and expected number of failures for a three month mission for pumps with mission completion ages of 36, 60, and 111 months. Also listed is the long-run average cost for a time-based replacement schedule with the associated pump ages. This table illustrates the trade-offs in cost and reliability that must be resolved for an effective maintenance policy. All

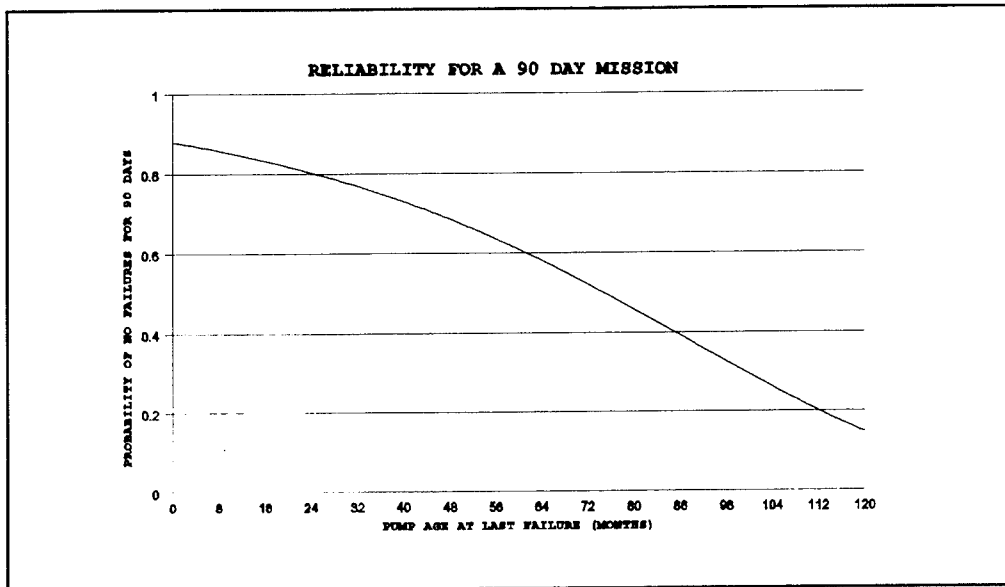


Figure 3.6: Reliability for a 90 Day Mission

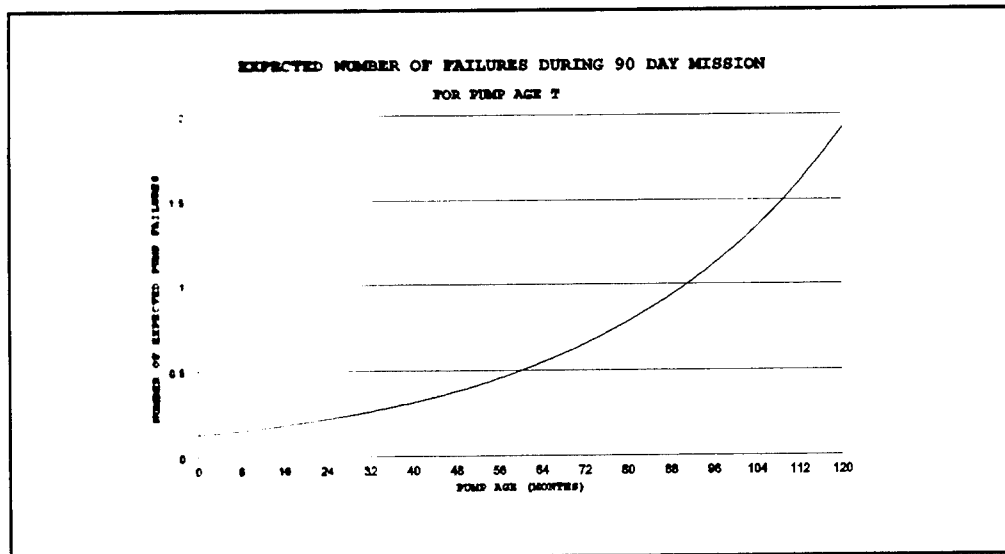


Figure 3.7: Expected Number of Failures During a 90 Day Mission as a Function of Pump Age

decision makers should understand the possible consequences of the proposed decisions. The use of probabilistic modeling is one way to evaluate the trade-offs and consequences of different maintenance policy.

TABLE 3.4: MISSION SURVIVABILITY AND EXPECTED FAILURES FOR A 3 MONTH MISSION

PUMP AGE AT MISSION COMPLETION (MONTHS)	PROBABILITY OF NO PUMP FAILURES	EXPECTED NUMBER OF PUMP FAILURES	LONG-RUN AVERAGE COST (\$/MONTH)
36	.76	.27	3674
60	.63	.46	2611
111	.23	1.47	2226

The calculations used in the preceding discussion are relatively simple to perform. Many similar calculations can also be made to address specific reliability requirements and different measures of effectiveness. The goal of this analysis has been to provide decision makers with a comprehensive assessment the cam-driven reciprocating pump's performance. Such information is necessary in the formulation of maintenance policy designed to address both economic and reliability concerns.

#### **IV. MODEL ASSESSMENT**

##### **A. VALIDITY OF THE DATA**

The formulation of a specific NHPP to model the failure characteristic of the cam-driven reciprocating pump was accomplished with a MLE calculation using actual failure data; data obtained primarily through the Navy's 3-M system. The accuracy of the model, therefore, depends largely upon the degree to which the data accurately represents actual system performance.

The current data collection method is far from perfect. The 3-M system itself suffers from many flaws. Part of the problem is inherent in the 3-M system, itself, and part is due to the fleet's attitude toward the system. When a failure occurs, the reporting process involve a crew member, normally junior enlisted personnel, filling out an OPNAV Form 4790/K. Some of the information to be included on the 4790/K are the equipment identification code, the date of failure, symptoms of failure, cause of failure, required repair parts, and required repair hours.

The 3-M system is designed to track failure data for everything from mechanical and electrical systems to fuel storage tanks. In doing so the system is so large and generic in reporting criteria that problems arise from a lack of standardization in recording specific failure information for individual pieces of equipment. This is often exemplified in general and nonspecific entries for failure symptoms and causes which can lead to confusion in reconstructing the actual equipment performance. The data can be further confounded by improperly entered identification codes for equipment and repair parts as well as incomplete entries. The 3-M system also does not capture all work performed by shipyard personnel during non-availability periods. All of these factors act to cloud the picture of true system performance and thus reduce the accuracy of any data analysis.

The data used in the formulation of this model may not be totally accurate in its portrayal of the cam-driven reciprocating pump's maintenance history, but it is currently the only viable source of data available for analysis.

## **B. VALIDITY OF THE ASSUMPTIONS**

In formulating the model, eight assumptions are made as to the characteristics of the cam-driven reciprocating pump

regarding failures, repairs and operation of the pump. Some of these assumptions are easy to accept; others require some discussion. First, this analysis assumes all pumps regardless of the submarine in which they are installed, experience roughly the same operating cycle. Pump operation during a submarine deployment will be similar between individual vessels. The deployment schedule, however, will differ between submarine platform and individual units. The long-run average deployment time is assumed to be the approximately the same for all submarines in the study. This, therefore, is a reasonable assumption.

The assumptions of independence between consecutive failures and repairs returning equipment to full operation are related. In reality these assumption are not always true. One failure can cause a subsequent failure at a later age. By the same token, the act of affixing repairs has been known to cause future failures. These failures may be totally unrelated to the previous failure, but were caused by improperly restoring the equipment. Likewise, a repair may not fully repair the problem and the pump is left in a condition below full operating capacity. This same argument can be made for a complete overhaul of the pump in that the overhauled pump is not as 'good-as-new'. Postrepair tests and

procedures, hopefully, identify and correct such faulty repairs. In regards to this data set, suspect failures following a repair were carefully scrutinized to catch such double-failures. In the end, however, all such dependence between failures and incomplete repairs cannot be sifted from the data set. Therefore, they will cause some loss of accuracy to the model.

### **C. ACCURACY OF THE MODEL**

. . .beware of mathematicians and all those who make empty prophecies. The danger already exists that the mathematicians have made a covenant with the devil to darken the spirit and to confine man in the bonds of Hell.

St. Augustine

St. Augustine may not have been talking about modelling the failure characteristics of a cam-driven reciprocating pump, but he does provide some wisdom for modelling in general. In fitting a mathematical model to characterize the failure behavior of any piece of equipment, it is naive to think that the model can perfectly predict the future performance. This would be true regardless of the system or the quality of the data. The results of this thesis, therefore, must be regarded in this light. This does not imply that any such model has no merit, but common sense and good engineering principles must be incorporated with any analytical results. The



incorporation of such information with the modelling results, provides a sound basis for making policy decisions.

The previous sections discussed several areas that may contribute to providing inaccuracies in the model, namely with the accuracy of the data and the validity of the assumptions. Another factor that must be noted is that the original data set only provides information on pump performance up to about 61 months. The majority of the observations actually occurs below 40 months. The model, however, extrapolates this information into performance predictions far and above this age. Any model inaccuracies will be magnified in this region. Also the use of this particular NHPP is only one parametric estimate of performance, many other more accurate models may exist.

Given all the model inaccuracies, does this model provide any information to policy makers? Yes! The formulation of this model and the associated data analysis have revealed several important features of the pump's failure characteristics. First, the pump exhibits wearout as noted by an increasing failure rate with age. Secondly, a time-based replacement decision of 60 months cannot be based solely on economic reasons. Thirdly, the model provides an estimate of the expected failures, failure times, and mission reliability

as the pump ages. This information, while not being totally precise, is pertinent to any decision maker evaluating maintenance policy.

## V. CONCLUSION AND RECOMMENDATIONS

As stated in the Introduction, the goal of this thesis has been to address the analytical needs of the Submarine Force necessary for a thorough assessment of current maintenance policies. Towards this end, an analysis of the failure characteristics of a cam-driven reciprocating pump has been conducted to demonstrate the degree of analysis required for a comprehensive assessment of equipment performance. Such analysis is necessary to assist decision makers in formulating maintenance policy especially when the Navy is faced with the reality of a shrinking budget. Decisions, however, must not be made strictly based on monetary measures. A thorough understanding of the consequences of any decision in terms of system performance and reliability is essential.

This analysis has been an extension of the work performed by Mr. Richard Youngk at SMMSO in evaluating the operation of the cam-driven reciprocating pump. His assistance and the support of SMMSO was vital in the completion of this work.

The results of this thesis are not perfect. This analysis used one particular model, namely a NHPP with a specific rate

function, to model failure characteristics. Other probabilistic models exist and a different model could possibly have provided a more accurate representation of pump performance. It could be advantageous to compare the results of this thesis with some of the other models. Additionally, one slight modification to the current model is recommended; perform a logarithmic transform on the failure times and recalculated the MLE. This would provide a smaller increase in the failure rate over time and may improve the model's fit.

As stated earlier, the accuracy of this model, and in general any model, is to a large degree reflected by the quality and quantity of data. The modelling process is not complete; as more information becomes available, the model should be updated. Likewise, future work should be devoted to improving the data collection system. If the 3-M system is to provide quality information, improvements must occur, specifically in the timeliness and accuracy of reporting performance data. Many independent data tracking programs have grown out of frustration with the current system. Such systems while providing quality information, place additional burdens on analyst and fleet personnel. The 3-M system needs to re-examine its purpose and assess how well it addresses the analytical needs of the Navy.

The evaluation of the cam-driven reciprocating pump should continue. New, quality data will improve the analysis and better quantify pump performance. Specifically, comprehensive cost and labor estimates will more clearly define the changing maintenance requirements of the pump as it ages.

This analysis has looked at only one small aspect of pump performance. Many additional areas merit research. One area centers around identifying differences in pump performance between platforms and individual submarines. The recognition and investigation of such differences could identify specific operating and maintenance practices unique to certain submarine that either enhance or degrade system performance.

Another area for further study involves analysis of the failure patterns by specific failure type. The thesis model considered failures as one entity when in fact many different failure types exist. If a pattern follows the occurrence of certain failures, then future failures and failure types could possibly be predicted based on previous failure. This would aid in planning maintenance, but more importantly it would steer investigation toward why such tendencies exist which could lead to equipment or procedural modifications.

The concept of failure prediction and prevention through monitoring also deserves attention. This again involves an analysis of failures by failure type, but additionally requires the identification of any precursors or indicators of impending failure. Such monitoring and predictive analysis already exist for many system. Further research could explore the value of certain precursors including determination of the expected time to failure given a specific precursors exist. This could aid in evaluating the effectiveness of monitoring procedures and possibly provide an optimal interval for planned monitoring.

One final area of research is that of time-based replacements for multiple systems. This concept was discussed briefly in Chapter III in which maintenance for different systems may be scheduled to coincide if significant interference removal or abnormal plant conditions are required for both. This thesis only addressed one system, but multiple systems could be analyzed similarly to determine the most effective replacement schedule to maintain the readiness of all systems involved.

This thesis has demonstrated the benefits of quantitative analysis and stochastic modelling in evaluating equipment performance. Such analysis are necessary to make the tough

decisions on maintenance policy in the wake of a shrinking budget. SMMSO, and specifically Mr. Richard Youngk, has taken on the task of incorporating these and similar concepts into current maintenance planning. It is hoped that policy makers will use these mathematical tool to make the well informed decisions necessary to maintain the U.S. Navy in top materiel readiness.

# APPENDIX A. PUMP FAILURE DATA

PUMP	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	T <sub>5</sub>	T <sub>6</sub>	FINAL EXCESS TOTAL		
							STATUS	LIFE	LIFE
1							C	43	43
2							C	41	41
3							C	21	21
4							C	10	10
5							C	32	32
6							C	13	13
7							C	20	20
8	1						C	9	10
9	2						C	27	29
10	2						C	22	24
11	2						C	24	26
12	3						C	11	14
13	4						C	9	13
14	5						C	22	27
15	8						C	2	10
16	10						R	0	10
17	11						R	0	11
18	13						C	1	14
19	13						R	0	13
20	15						C	8	23
21	17						R	0	17
22	18						C	7	25
23	26						C	7	33
24	27						R	0	27
25	29						C	7	36
26	8	10					C	11	21
27	4	14					C	8	22
28	10	14					R	0	14
29	1	18					C	34	52
30	17	21					C	19	40
31	19	24					C	13	37
32	22	28					R	0	28
33	23	31					C	22	53



PUMP	FINAL EXCESS TOTAL						STATUS	LIFE	LIFE
	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	T <sub>5</sub>	T <sub>6</sub>			
34	14	34					C	16	50
35	16	38					R	0	38
36	18	41					C	18	59
37	26	61					C	4	65
38	11	12	15				C	11	26
39	4	6	21				C	15	36
40	5	20	23				C	30	53
41	10	17	26				R	0	26
42	17	32	33				C	3	36
43	36	37	41				C	3	44
44	16	23	43				R	0	43
45	17	25	43				C	1	44
46	18	24	56				C	7	63
47	2	8	9	10			R	0	10
48	15	17	22	25			C	1	26
49	15	32	38	39			C	17	56
50	27	32	37	42			R	0	42
51	3	27	42	48			C	2	50
52	4	19	52	52			C	8	60
53	6	15	17	20	24		C	0	24
54	9	12	16	26	26		C	3	29
55	4	6	14	25	34		C	3	37
56	5	16	18	36	43		C	2	45
57	4	10	25	28	44		R	0	44
58	19	31	42	46	49		C	4	53
59	12	36	43	44	51		C	11	62
60	11	22	25	34	35	37	R	0	37
61	13	17	30	32	36	40	C	13	53

**LEGEND:**

T<sub>i</sub> = FAILURE TIME, IN MONTHS, FOR FAILURE NUMBER i

C INDICATES PUMP LAST OBSERVED OPERATING

R INDICATES PUMP WAS REPLACED AT LAST OBSERVATION

**NOTES:**

ALL TIMES ARE ROUNDED TO THE NEAREST MONTH

TOTAL FAILURES: 140, TOTAL REPLACEMENTS: 14

TOTAL EXPOSURE: 2020

## APPENDIX B. DERIVATION OF THE MAXIMUM LIKELIHOOD ESTIMATOR

Let

$k$  = index for the number of pumps in the sample set

$j$  = index for the individual pumps,  $j = 1, 2, \dots, k$

$i$  = index for the failure number for pump  $j$ ,  $i = 1, 2, \dots, n_j$

$n_j$  = the number of failures for pump  $j$

$f_j$  = time of the last observation for pump  $j$

\*Note: time is equivalent to pump age in this analysis\*

$t_{i,j}$  = time of failure  $i$  for pump  $j$

$p(t_{i,j})$  = probability that the failure occurring at time  $t_{i,j}$  is repairable.

$q(t_{i,j})$  = probability that the failure occurring at time  $t_{i,j}$  is not repairable and therefore requires pump replacement.  $q(t_{i,j}) = 1 - p(t_{i,j})$ .

$F_j$  = indicator variable indicating status of the last observation

$F_j = 1$  for non-repairable failure

$F_j = 0$  for pump last observed operating

$\lambda(t_{i,j})$  = rate of occurrence of pump failures for pump age  $t_{i,j}$ . The failure may or may not be repairable.

$\Lambda(t_{i,j})$  = the integrated value of the failure rate from pump installation to age  $t_{i,j}$ ; the expected number of failures (repairable) up to age  $t_{i,j}$

$\theta$  = vector of parameters,  $\theta = (\alpha, \beta)$

$L(\theta)$  = Combined Likelihood Function for pump failures

$L_c(\theta)$  = Conditional Likelihood Function for pump failures

NOTE: If the last observation is a non-repairable failure then  $f_j = t_{n_j,j}$  and  $F_j = 1$ .

Consider the likelihood function for one pump, i.e., the  $j^{\text{th}}$ :

$$L_j(\theta) = e^{-\Lambda(t_{1j})} \lambda(t_{1j}) p(t_{1j}) e^{-[\Lambda(t_{2j}) - \Lambda(t_{1j})]} \lambda(t_{2j}) p(t_{2j}) \dots$$

$$e^{-[\Lambda(t_{nj}) - \Lambda(t_{nj-1})]} \left( \frac{q(t_{nj})}{p(t_{nj})} \right)^{F_j}; \quad (\text{B1})$$

Note that in (B1) the dependency of  $\Lambda$ ,  $\lambda$  and  $p$  on parameters  $\theta$  is suppressed.

For the case with the last observation being a non-repairable failure,  $f_j = t_{nj,j}$  and  $F_j = 1$  so expression (B1) reduces to :

$$L_j(\theta) = e^{-\Lambda(t_{1j})} \lambda(t_{1j}) p(t_{1j}) e^{-[\Lambda(t_{2j}) - \Lambda(t_{1j})]} \lambda(t_{2j}) p(t_{2j}) \dots$$

$$e^{-[\Lambda(t_{nj}) - \Lambda(t_{nj-1})]} \lambda(t_{nj}) q(t_{nj}). \quad (\text{B2})$$

Equation (B2) simplifies to:

$$L_j(\theta) = e^{-\Lambda(t_{nj})} \lambda(t_{1j}) p(t_{1j}) \lambda(t_{2j}) p(t_{2j}) \dots \lambda(t_{nj}) p(t_{nj}) \left( \frac{q(t_{nj})}{p(t_{nj})} \right)^{F_j}$$

$$= e^{-\Lambda(t_{nj})} \prod_{i=1}^{n_j} \lambda(t_{ij}) p(t_{ij}) \left( \frac{q(t_{nj})}{p(t_{nj})} \right)^{F_j}. \quad (\text{B3})$$

Combining the observations from all pumps:

$$L(\theta) = e^{-\sum_{j=1}^k \Lambda(f_j)} \prod_{j=1}^k \prod_{i=1}^{n_j} \lambda(t_{ij}) \prod_{j=1}^k \left( \frac{q(t_{nj})}{p(t_{nj})} \right)^{F_j} \prod_{j=1}^k \prod_{i=1}^{n_j} p(t_{ij}). \quad (\text{B4})$$

To obtain the conditional probability density function, or pdf, of the observations given  $n_j$ ,  $j=1,2,\dots,k$ , failures, divide  $L(\theta)$  by

$$\frac{\prod_{j=1}^k \Lambda(f_j)^{n_j} e^{-\sum_{j=1}^k \Lambda(f_j)}}{\prod_{j=1}^k n_j!} \prod_{j=1}^k \prod_{i=1}^{n_j} \lambda(t_{ij}) \prod_{j=1}^k \left( \frac{q(t_{nj})}{p(t_{nj})} \right)^{F_j} \prod_{j=1}^k \prod_{i=1}^{n_j} p(t_{ij}).$$

This results in the conditional likelihood function:

$$L_c(\theta) = \frac{\prod_{j=1}^k \prod_{i=1}^{n_j} \lambda(t_{ij}) \prod_{j=1}^k n_j!}{\prod_{j=1}^k \Lambda(f_j)^{n_j}} \quad (\text{B5})$$

### SPECIAL CASE: MODEL FOR INCREASING HAZARD

In the remainder of this thesis we consider the following specific model, developed by Cox and Lewis (1966), pp. 45-54:

$$\lambda(t_{ij}) = e^{\alpha \cdot \beta t_{ij}} \quad \Lambda(t_{ij}) = \frac{e^{\alpha}}{\beta} (e^{\beta t_{ij}} - 1).$$

Then the combined likelihood function,  $L(\theta)$ , becomes:

$$L(\theta) = e^{\left[ \sum_{j=1}^k n_j \cdot \beta \sum_{i=1}^{n_j} t_{ij} - \sum_{j=1}^k \frac{e^{\alpha}}{\beta} (e^{\beta t_{ij}} - 1) \right]} \prod_{j=1}^k \left( \frac{q(t_{nj})}{p(t_{nj})} \right)^{F_j} \prod_{j=1}^k \prod_{i=1}^{n_j} p(t_{ij}). \quad (\text{B6})$$

Similarly the conditional likelihood function,  $L_c(\theta)$ , is

$$L_c(\theta) = \frac{e^{\beta \sum_{j=1}^k \sum_{i=1}^{n_j} t_{ij}} \prod_{j=1}^k \beta^{n_j} \prod_{j=1}^k n_j!}{\prod_{j=1}^k (e^{\beta t_{ij}} - 1)^{n_j}} \quad (\text{B7})$$

Here  $\theta = (\alpha, \beta)$  and any parameters used to specify  $p(t_{i,j})$ .

Taking the logarithm of  $L_c(\theta)$  results in

$$\log L_c(\theta) = \beta \sum_{j=1}^k \sum_{i=1}^{n_j} t_{ij} + \sum_{j=1}^k \log(n_j!) + \log \beta \sum_{j=1}^k n_j - \sum_{j=1}^k n_j \log(e^{\beta t_{ij}} - 1) \quad (\text{B8})$$

To find the Maximum Likelihood Estimator for  $\beta$ , take the derivative of  $\log L_c(\theta)$ , equation (B8), with respect to  $\beta$ , set it equal to zero, and solve for  $\beta$ . The resulting derivative is

$$\frac{d(\log L_c(\theta))}{d\beta} = \sum_{j=1}^k \sum_{i=1}^{n_j} t_{ij} + \frac{1}{\beta} \sum_{j=1}^k n_j - \sum_{j=1}^k \frac{n_j f_j}{(1 - e^{-\beta f_j})} \quad (\text{B9})$$

Graphing Equation (B9) over a range of values for  $\beta$  and visually locating the zero intercept produces a rough estimate of the maximizing value for  $\beta$ . This initial approximation can be further refined by using a numerical method, such as Newton's Method, to solve for the root. This reduces the mathematical complexity by eliminating the need for deriving a closed form solution for the root if a closed form solution does indeed exist.

A similar method is used to find the MLE of  $\alpha$ . The Conditional Likelihood Function,  $L_c(\theta)$ , however, cannot be used to solve for  $\alpha$  as it was for  $\beta$  since the conditioning process removes  $\alpha$  as a parameter. Therefore,  $L(\theta)$ , Equation (B6) must be used to find the maximizing value of  $\alpha$ . Taking the logarithm of  $L(\theta)$ , differentiating with respect to  $\alpha$ ,

and setting the equation equal to zero results in the following expression:

$$\frac{d[\log L(\theta)]}{d\alpha} = \sum_{j=1}^k n_j - \frac{e^{\alpha}}{\beta} \sum_{j=1}^k (e^{\beta f_{j-1}}) = 0, \quad (\text{B10})$$

Solving the equation in terms of  $\alpha$  results in the following expression:

$$\hat{\alpha} = \ln \left[ \frac{\hat{\beta} \sum_{j=1}^k n_j}{\sum_{j=1}^k (e^{\beta f_{j-1}})} \right]. \quad (\text{B11})$$

Inserting the maximizing value of  $\beta$  into Equation (B11) produces the resulting maximizing value of  $\alpha$ .

#### **CONFIDENCE BOUNDS FOR $\alpha$ AND $\beta$**

The confidence bounds for the estimation of  $\alpha$  and  $\beta$  are derived using the Fisher information matrix to obtain the asymptotic variances and covariances of the MLE's. The Fisher information matrix is the composed of the negative second partial derivatives of the sample log likelihood. Inverting the Fisher information matrix evaluated at the MLE

for  $\alpha$  and  $\beta$  produces an estimate of the variance/covariance matrix. The variance can then be used to derive the confidence bounds for the MLE's.

The partial derivatives are derived using the log of equation (B6) and are as follows:

$$\frac{\partial^2 \log L(\theta)}{\partial \alpha^2} = -\frac{e^\alpha}{\beta} \sum_{j=1}^k (e^{\beta f_{j-1}}) \quad (\text{B12})$$

$$\frac{\partial^2 \log L(\theta)}{\partial \alpha \partial \beta} = \frac{\partial^2 \log L(\theta)}{\partial \beta \partial \alpha} = -\frac{e^\alpha}{\beta^2} \sum_{j=1}^k (e^{\beta f_{j-1}}) - \frac{e^\alpha}{\beta} \sum_{j=1}^k (f_j e^{\beta f_j}) \quad (\text{B13})$$

$$\frac{\partial^2 \log L(\theta)}{\partial \beta^2} = -\frac{2e^\alpha}{\beta^3} \sum_{j=1}^k (e^{\beta f_{j-1}}) + \frac{2e^\alpha}{\beta^2} \sum_{j=1}^k (f_j e^{\beta f_j}) - \frac{e^\alpha}{\beta} \sum_{j=1}^k (f_j^2 e^{\beta f_j}). \quad (\text{B14})$$



## APPENDIX C. MATHCAD 3.1 PROGRAMS

\*\*\*\*\*

# MATHCAD 3.1 PROGRAM TO COMPUTE MLE OF BETA

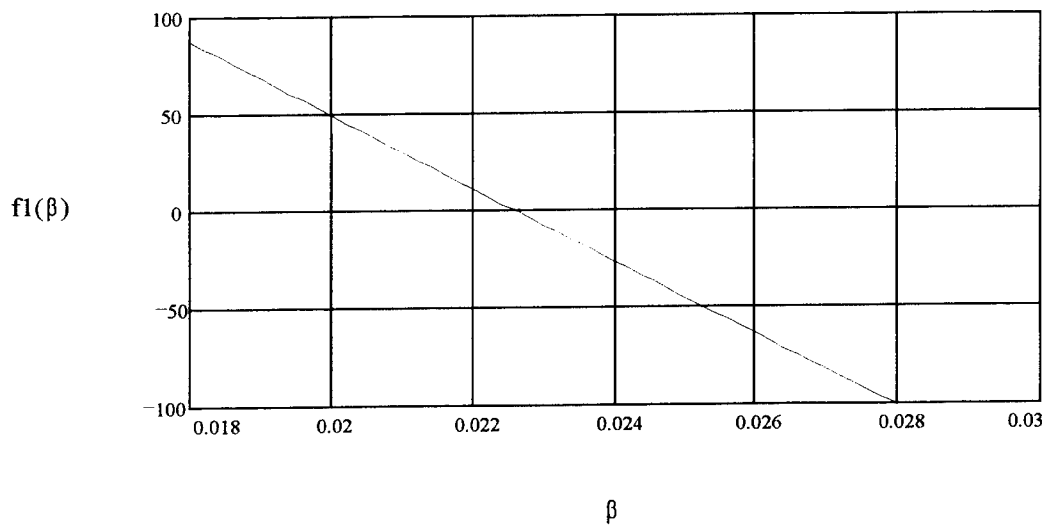
\*\*\*\*\*

j := 0..60 k := 1..150

$n_j := \text{READ}(\text{NU})$      $tt_j := \text{READ}(\text{TTU})$      $f_j := \text{READ}(\text{TFU})$      $\beta_k := \frac{k}{5000}$

$$f1(\beta) := \left( \sum_j tt_j + \frac{1}{\beta} \cdot \sum_j n_j \right) - \sum_j \frac{n_j \cdot f_j}{(1 - e^{-\beta \cdot f_j})} \quad f2(\beta) := \sum_j \frac{n_j \cdot (f_j)^2 \cdot e^{-\beta \cdot f_j}}{(1 - e^{-\beta \cdot f_j})^2} - \frac{1}{\beta^2} \cdot \sum_j n_j$$

THE DERIVATIVE OF THE CONDITIONAL LOG LIKELIHOOD  
PLOTTED OVER VALUES OF BETA



## NEWTON'S METHOD

N := 60    i := 0..N    err := 10<sup>-8</sup>

$y_0 := .023$      $f1(y_0) = -7.88315$

$$y_{i+1} := \text{until} \left[ \left| f1(y_i) \right| - \text{err}, \left( y_i - \frac{f1(y_i)}{f2(y_i)} \right) \right]$$

$$y = \begin{bmatrix} 0.023 \\ 0.0225818402 \\ 0.0225823081 \\ 0.0225823081 \\ 0 \end{bmatrix}$$

$n2 := \text{last}(y) - 1$

$n2 = 3$

$f1(y_{n2}) = 0$

$y_{n2} = 0.02258$

\*\*\*\*\*  
**MATHCAD 3.1 PROGRAM TO DERIVE THE 95% CI FOR MLE'S ALPHA AND BETA**  
 \*\*\*\*\*

j := 0..60

n<sub>j</sub> := READ(NU)      tt<sub>j</sub> := READ(TTU)      f<sub>j</sub> := READ(TFU)

Second partial derivatives of the unconditioned MLF

$$daa(a, b) := -\frac{e^a}{b} \sum_j \left( e^{\frac{b \cdot f_j}{b}} - 1 \right) \quad dab(a, b) := \frac{e^a}{b^2} \sum_j \left( e^{\frac{b \cdot f_j}{b}} - 1 \right) - \frac{e^a}{b} \sum_j \left[ \left( e^{\frac{b \cdot f_j}{b}} \right) \cdot f_j \right]$$

$$dbb(a, b) := \frac{-2 \cdot e^a}{b^3} \left[ \sum_j \left( e^{\frac{b \cdot f_j}{b}} - 1 \right) \right] + \frac{e^a}{b^2} \sum_j \left[ \left( e^{\frac{b \cdot f_j}{b}} \right) \cdot f_j \right] + \frac{e^a}{b^2} \left[ \sum_j \left[ \left( e^{\frac{b \cdot f_j}{b}} \right) \cdot f_j \right] \right] - \frac{e^a}{b} \sum_j \left[ \left( e^{\frac{b \cdot f_j}{b}} \right) \cdot (f_j)^2 \right]$$

alpha := -3.189

beta := .02258

daa(alpha, beta) = -139.441

dbb(alpha, beta) = -1.257 · 10<sup>5</sup>

dab(alpha, beta) = -3.56 · 10<sup>3</sup>

$$MAT := \begin{pmatrix} -daa(alpha, beta) & -dab(alpha, beta) \\ -dab(alpha, beta) & -dbb(alpha, beta) \end{pmatrix}^{-1}$$

$$MAT = \begin{pmatrix} 0.026 & -7.335 \cdot 10^{-4} \\ -7.335 \cdot 10^{-4} & 2.873 \cdot 10^{-5} \end{pmatrix}$$

**95% CONFIDENCE INTERVAL FOR ALPHA**

$$LLA := alpha - 1.96 \cdot (MAT_{0,0})^{.5}$$

LLA = -3.504

$$ULA := alpha + 1.96 \cdot (MAT_{0,0})^{.5}$$

ULA = -2.874

**95% CONFIDENCE INTERVAL FOR BETA**

$$LLB := beta - 1.96 \cdot (MAT_{1,1})^{.5}$$

LLB = 0.012

$$ULB := beta + 1.96 \cdot (MAT_{1,1})^{.5}$$

ULB = 0.033

\*\*\*\*\*  
**MATHCAD 3.1 PROGRAM TO DETERMINE THE OPTIMAL REPLACEMENT INTERVAL**  
 \*\*\*\*\*

**Expected life of the pump (ELife):**

**T = timed replacement interval in months**

**p = probability that a failure is repairable**

**q = probability that a failure is not repairable and requires pump replacement**

$i = 1..200 \quad T_i := i$

$a := -3.189 \quad b := 0.02258 \quad p := \frac{126}{140} \quad q := 1 - p$

$$\Lambda(t) := \frac{e^a}{b} \cdot (e^{b \cdot t} - 1) \quad d\Lambda(t) := e^{a+b \cdot t}$$

**tff = case in which pump is replace before scheduled interval T**

**ttT = scheduled replacement at time interval T**

$$tff_i := \int_0^{T_i} q \cdot x \cdot e^{-\Lambda(x) \cdot q} \cdot d\Lambda(x) \, dx \quad ttT_i := T_i \cdot e^{-\Lambda(T_i) \cdot q} \quad f(x) := q \cdot x \cdot e^{-\Lambda(x) \cdot q} \cdot d\Lambda(x)$$

$$ELife_i := tff_i + ttT_i$$

**Expected reward/cost :**

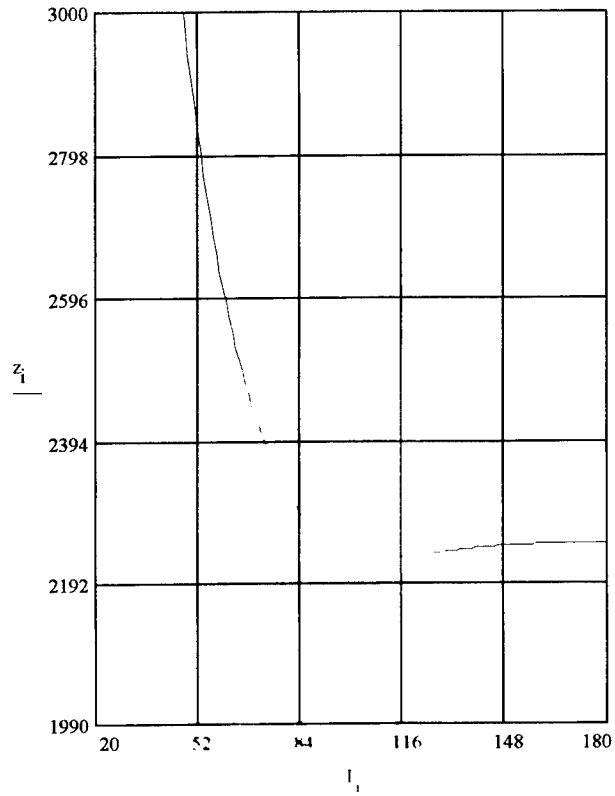
**CReplace := 111000**

$$CRepair := 4900$$

$$ECost_i := CReplace + CRepair \cdot \left( \Lambda(T_i) \cdot p \cdot e^{-\Lambda(T_i) \cdot q} + \frac{p}{q} - \Lambda(T_i) \cdot p \cdot e^{-\Lambda(T_i) \cdot q} - \frac{p}{q} \cdot e^{-\Lambda(T_i) \cdot q} \right)$$

# **Reward / Renewal Process: Long Run Average Reward (z):**

$$z_i := \frac{ECost_i}{ELife_i}$$



$$\min_0 := 10000000$$

$$z_0 := 1000000000000$$

$$j := 1..300$$

$$\min_j := \text{until}[(z_j - z_{j+1}), (z_j)]$$

$$n2 := \text{last}(\min)$$

$$z_{n2-1} = 2.22645726 \cdot 10^3$$

$$z_{n2} = 2.22641167 \cdot 10^3$$

$$z_{n2+1} = 2.2264619 \cdot 10^3$$

$$n2 = 111$$

\*\*\*\*\*

### MATHCAD 3.1 PROGRAM TO SIMULATE PUMP FAILURE TIMES

\*\*\*\*\*

Simulation of failure times using method of Law & Kelton, 1982, pp. 509-510.

ORIGIN:=0      n:=500      a:=3.189      b:=.02258

i:=1..14      j:=1..n      tp<sub>0,j</sub>:=0      r<sub>j</sub>:=1

u<sub>i,j</sub>:=rnd(r<sub>j</sub>)

tp<sub>i,j</sub>:=tp<sub>(i-1),j</sub> - ln(u<sub>i,j</sub>)

$$t_{i,j} := \frac{1}{b} \cdot \ln \left[ \frac{e^{-a}}{\left(\frac{1}{b}\right)} \cdot tp_{i,j} + 1 \right]$$

fail1 := (t<sup>T</sup>)<sup><1></sup>

fail2 := (t<sup>T</sup>)<sup><2></sup>

fail3 := (t<sup>T</sup>)<sup><3></sup>

fail4 := (t<sup>T</sup>)<sup><4></sup>

fail5 := (t<sup>T</sup>)<sup><5></sup>

fail6 := (t<sup>T</sup>)<sup><6></sup>

fail7 := (t<sup>T</sup>)<sup><7></sup>

fail8 := (t<sup>T</sup>)<sup><8></sup>

fail5 := (t<sup>T</sup>)<sup><5></sup>

fail6 := (t<sup>T</sup>)<sup><6></sup>

fail7 := (t<sup>T</sup>)<sup><7></sup>

fail8 := (t<sup>T</sup>)<sup><8></sup>

fail9 := (t<sup>T</sup>)<sup><9></sup>

fail10 := (t<sup>T</sup>)<sup><10></sup>

fail11 := (t<sup>T</sup>)<sup><11></sup>

fail12 := (t<sup>T</sup>)<sup><12></sup>

fail13 := (t<sup>T</sup>)<sup><13></sup>

fail14 := (t<sup>T</sup>)<sup><14></sup>

### CALCULATION OF MEAN TIME TO FAILURE FOR FAILURE i

$$avg_1 := \frac{\sum_{j=1}^{42} fail1_j}{n}$$

$$avg_2 := \frac{\sum_{j=1}^{42} fail2_j}{n}$$

$$avg_3 := \frac{\sum_{j=1}^{42} fail3_j}{n}$$

$$avg_{10} := \frac{\sum_{j=1}^{42} fail10_j}{n}$$

$$avg_{11} := \frac{\sum_{j=1}^{42} fail11_j}{n}$$

$$avg_4 := \frac{\sum_{j=1}^{42} fail4_j}{n}$$

$$avg_5 := \frac{\sum_{j=1}^{42} fail5_j}{n}$$

$$avg_6 := \frac{\sum_{j=1}^{42} fail6_j}{n}$$

$$avg_{12} := \frac{\sum_{j=1}^{42} fail12_j}{n}$$

$$avg_{13} := \frac{\sum_{j=1}^{42} fail13_j}{n}$$

$$avg_7 := \frac{\sum_{j=1}^{42} fail7_j}{n}$$

$$avg_8 := \frac{\sum_{j=1}^{42} fail8_j}{n}$$

$$avg_9 := \frac{\sum_{j=1}^{42} fail9_j}{n}$$

$$avg_{14} := \frac{\sum_{j=1}^{42} fail14_j}{n}$$

avg<sup>T</sup> = ( 0   16.668   29.288   39.841   48.154   55.474   61.885   67.59   72.539   76.899   81.285   84.829  
(88.276) (91.262) (94.209)

# CALCULATION OF THE STANDARD ERROR FOR THE SAMPLE MEAN TIME TO FAILURE i

j := 1..n

$$\text{samplestd}_1 := \left[ \frac{\sum_j (\text{fail1}_j - \text{avg}_1)^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_2 := \left[ \frac{\sum_j (\text{fail2}_j - \text{avg}_2)^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_3 := \left[ \frac{\sum_j (\text{fail3}_j - \text{avg}_3)^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_4 := \left[ \frac{\sum_j (\text{fail4}_j - \text{avg}_4)^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_5 := \left[ \frac{\sum_j (\text{fail5}_j - \text{avg}_5)^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_6 := \left[ \frac{\sum_j (\text{fail6}_j - \text{avg}_6)^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_7 := \left[ \frac{\sum_j (\text{fail7}_j - \text{avg}_7)^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_8 := \left[ \frac{\sum_j (\text{fail8}_j - \text{avg}_8)^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_9 := \left[ \frac{\sum_j (\text{fail9}_j - \text{avg}_9)^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_{10} := \left[ \frac{\sum_j (\text{fail10}_j - \text{avg}_{10})^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_{11} := \left[ \frac{\sum_j (\text{fail11}_j - \text{avg}_{11})^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_{12} := \left[ \frac{\sum_j (\text{fail12}_j - \text{avg}_{12})^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_{13} := \left[ \frac{\sum_j (\text{fail13}_j - \text{avg}_{13})^2}{n - 1} \right]^{.5}$$

$$\text{samplestd}_{14} := \left[ \frac{\sum_j (\text{fail14}_j - \text{avg}_{14})^2}{n - 1} \right]^{.5}$$

$$\text{stderror} := \left[ \frac{1}{(\sqrt{n})} \right] \cdot \text{samplestd}$$

stderror<sup>T</sup> = ( 0 0.602 0.672 0.691 0.681 0.659 0.634 0.624 0.589 0.555 0.539 0.517 0.501  
(0.489) (0.479)

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3.	Dr. Peter Purdue (Code OR/Pd) Naval Postgraduate School Monterey CA 93943-5000	1
4.	Dr. Donald Gaver (Code OR/Gv) Naval Postgraduate School Monterey CA 93943-5000	1
5.	Dr. Patricia Jacobs (Code OR/Jc) Naval Postgraduate School Monterey CA 93943 5000	1
6.	Naval Sea Systems Command Commander, PMS 90 2531 Jefferson Davis Highway Arlington VA 22242-5160	3
7.	Naval Sea Systems Command Commander, SEA 81 2531 Jefferson Davis Highway Arlington VA 22242-5160	1
8.	Naval Sea Systems Command Commander, IMS 92T 2531 Jefferson Davis Highway Arlington VA 22242-5160	1
9.	Naval Sea Systems Command Commander, SEA 03 2531 Jefferson Davis Highway Arlington VA 22242-5160	1
10.	LT Donald D. Dudenhoeffer Naval Postgraduate School 2 University Circle Monterey CA 93943-1312	1